
Supplementary information

Insights into lattice thermal transport mechanisms in layered chalcogenides $X_2\text{Pd}Y_6$ ($X = \text{Nb, Ta}$; $Y = \text{S, Se}$) via machine learning molecular dynamics simulations

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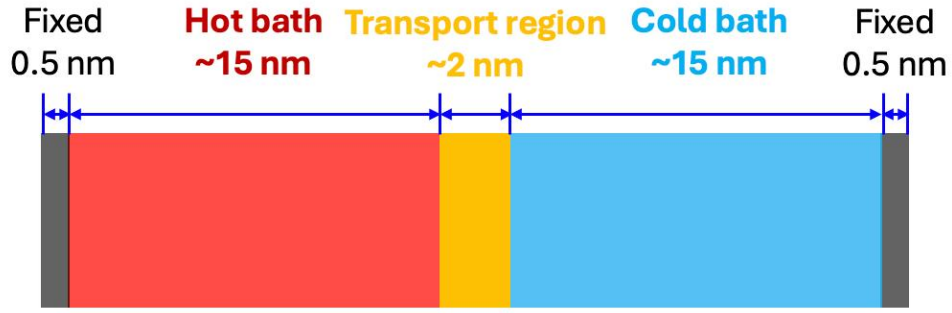


Figure S1. Schematic setup for NEMD simulations.

The NEMD simulation setup is schematically illustrated in Fig. S1. The two terminal regions, each with a length of 0.5 nm, were fixed. The heat-source and heat-sink regions had lengths of 15.0 nm. The spectral heat current was evaluated in the central transport region, whose length was ~ 2 nm. Periodic boundary conditions were applied along transverse directions.

Each system was equilibrated for 200 ps in the NPT ensemble and subsequently for 200 ps in the NVT ensemble, using a 1 fs time step. An NEMD simulation was then performed for 1 ns using Langevin heat baths to build the non-equilibrium steady state. Subsequently, another 1 ns of NEMD simulation was performed for data collection (temperature, heat flux, and spectral decomposition). The low average thermostat temperature of 50 K was used to obtain ballistic transport, with a temperature bias of 10 K. The thermostat coupling time was 50 fs.

To assess convergence, we divided each production trajectory into 2 consecutive blocks and verified that the integrated and spectral conductance were stable to within 5%. The reported $G(\omega)$ is the average over three independent trajectories initialized with different velocity seeds.

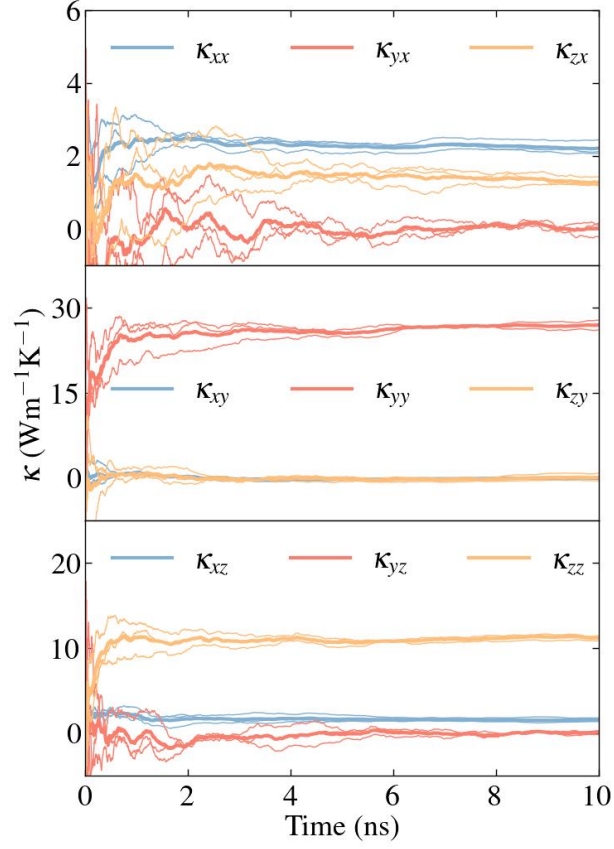


Figure S2. Full tensor calculation of Ta_2PdS_6 lattice thermal conductivity (LTC) at 300K.

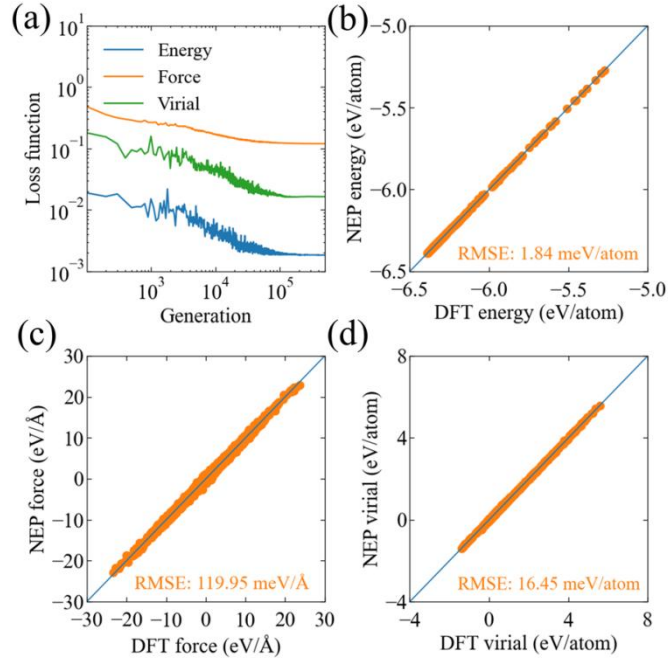


Figure S3. (a) Nb_2PdS_6 NEP model training iterative process of energy, force and viria; (b)-(d) Parity plot of energy, force and virial.

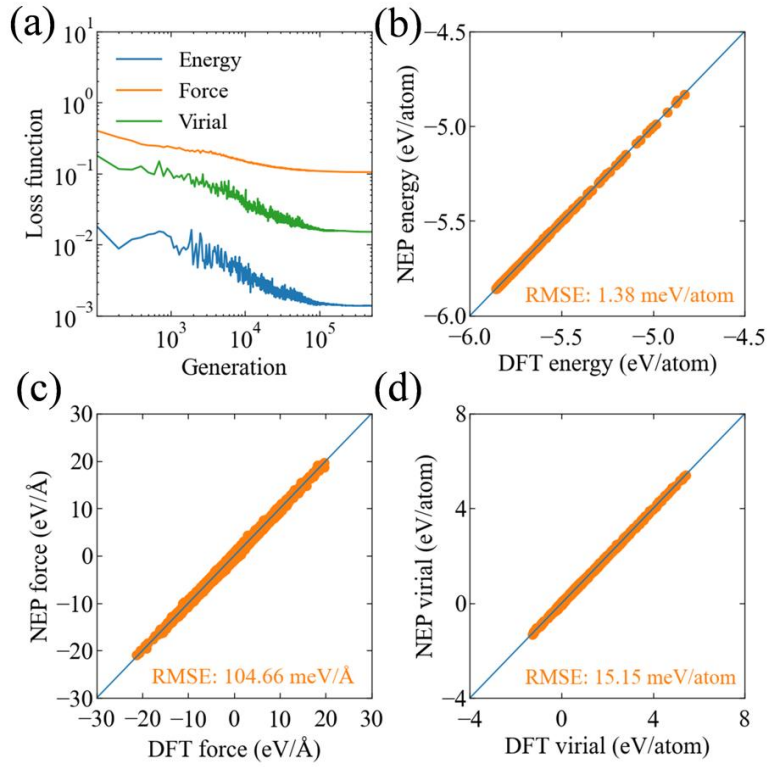


Figure S4. (a) Nb₂PdSe₆ NEP model training iterative process of energy, force and virial; (b)-(d) Parity plot of energy, force and virial.

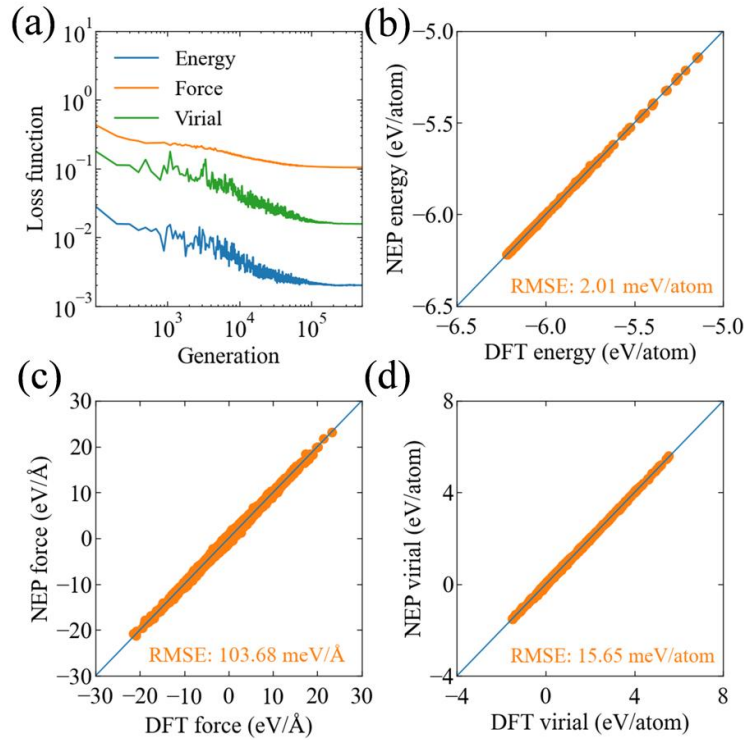


Figure S5. (a) Ta₂PdSe₆ NEP model training iterative process of energy, force and virial; (b)-(d) Parity plot of energy, force and virial.

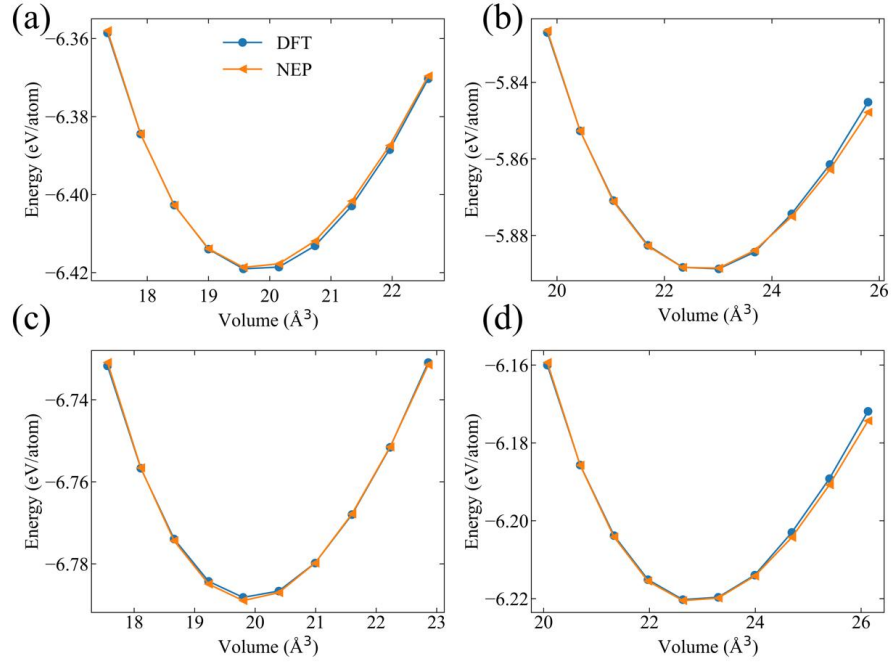


Figure S6. Predicted NEP and DFT-calculated energy-volume relationships for (a) Nb_2PdS_6 ; (b) Nb_2PdSe_6 ; (c) Ta_2PdS_6 ; (d) Ta_2PdSe_6 .

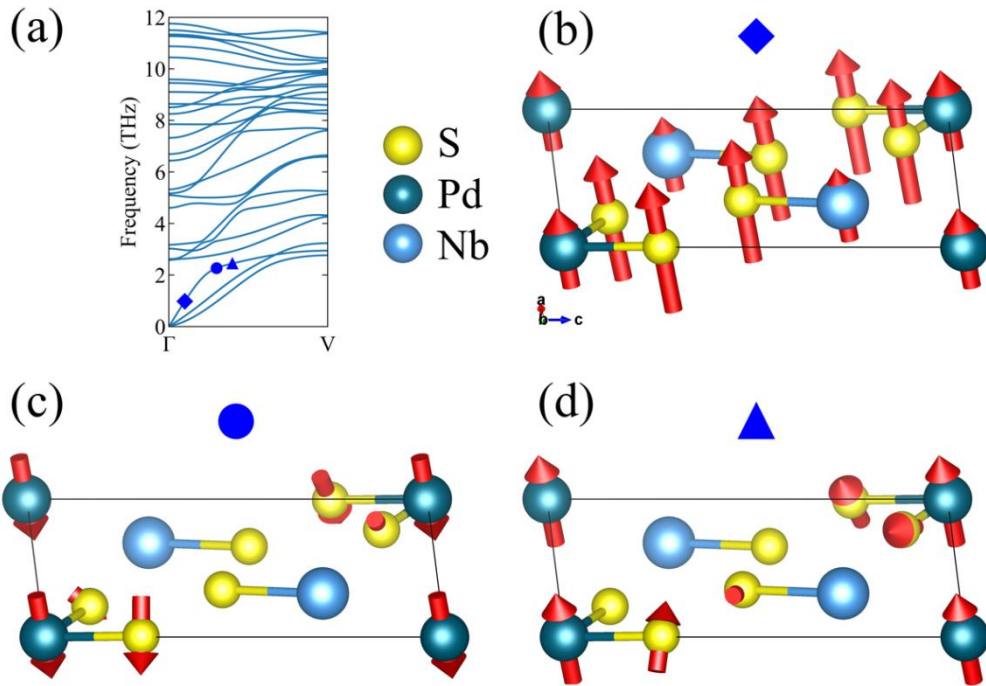


Figure S7. (a) Phonon dispersion curve of Nb_2PdS_6 . Visualization of phonon vibration modes marked in the dispersion curve: (b) Diamond; (c) Circle; (d) Triangle.

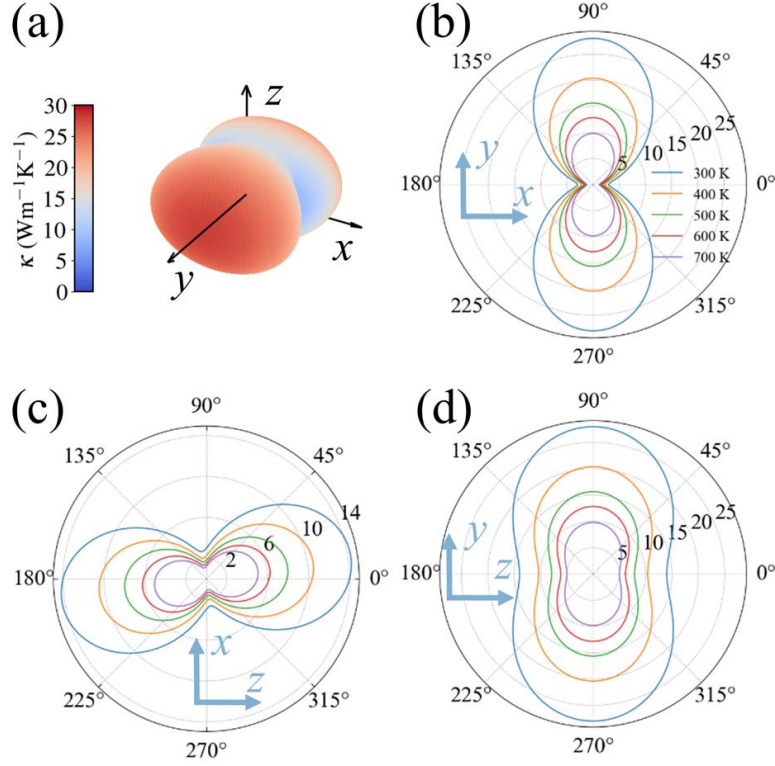


Figure S8. (a) Polar plot of LTC for Nb₂PdS₆ at 300K; (b)-(d) Two-dimensional projection maps of LTC on the crystal planes {001} (b), {010} (c), and {100} (d). The units for LTC are all Wm⁻¹K⁻¹.

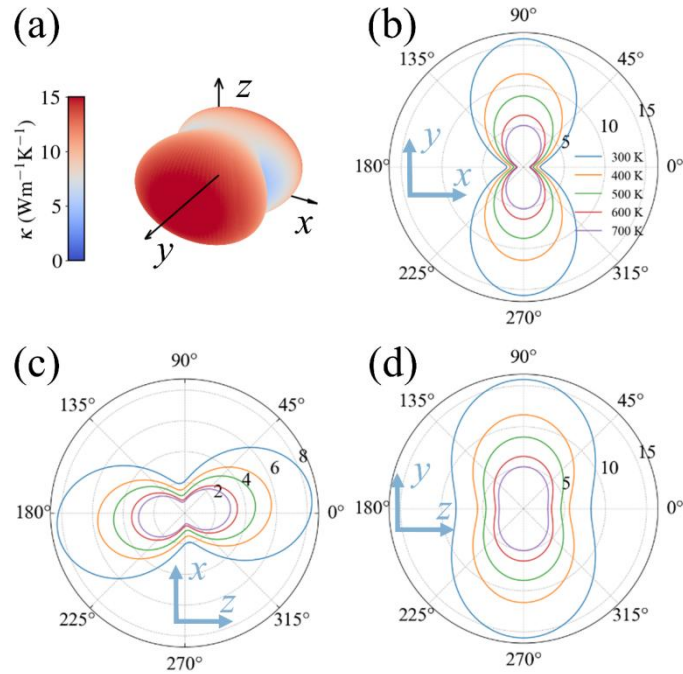


Figure S9. (a) Polar plot of LTC for Nb₂PdSe₆ at 300K. (b) - (d) Two-dimensional projection maps of LTC on the crystal planes {001} (b), {010} (c), and {100} (d). The units for LTC are all Wm⁻¹ K⁻¹.

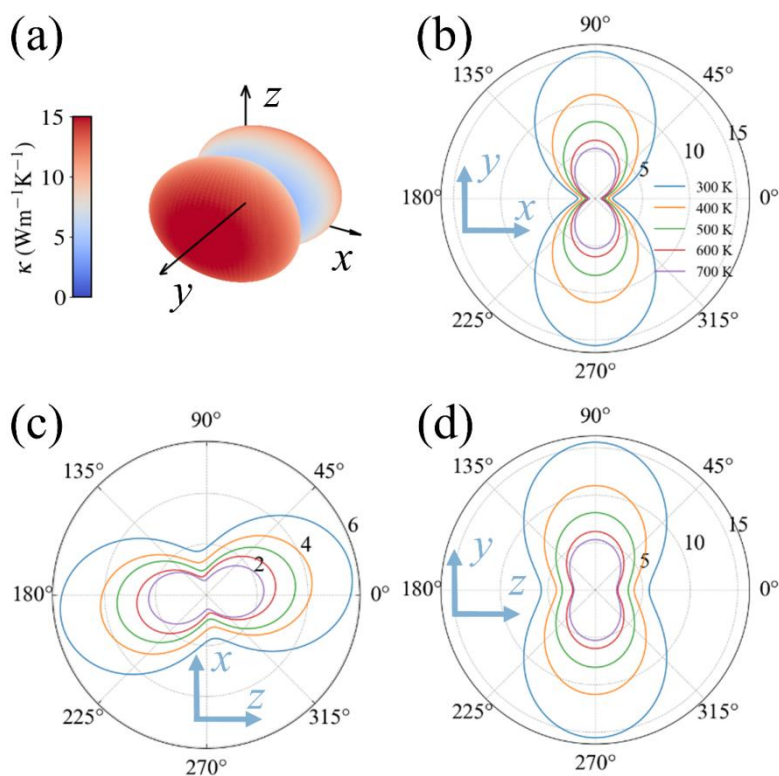


Figure S10. (a) Polar plot of LTC for Ta₂PdSe₆ at 300K. (b) - (d) Two-dimensional projection maps of LTC on the crystal planes {001} (b), {010} (c), and {100} (d). The units for LTC are all W m⁻¹ K⁻¹.

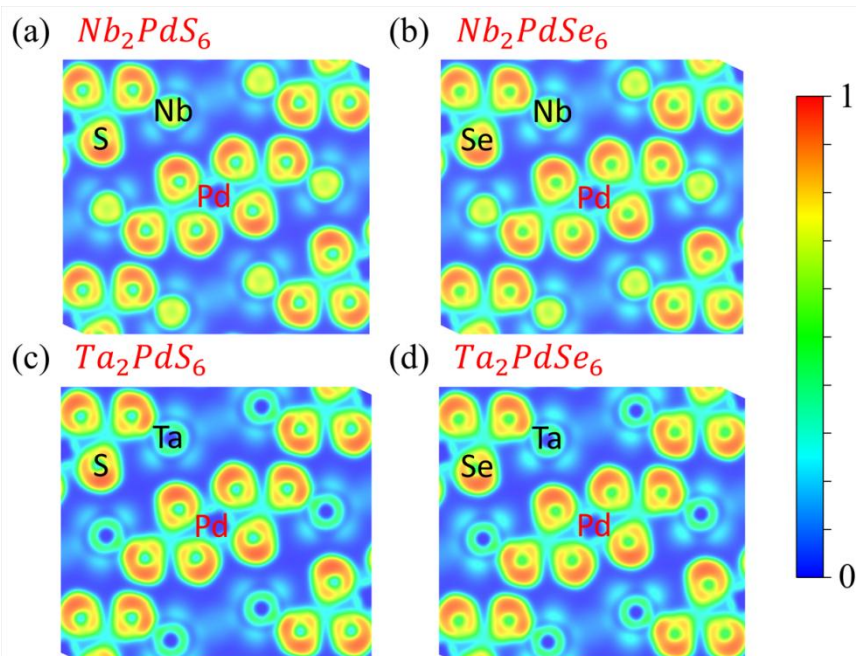


Figure S11. Distribution of the electron localization function (ELF) for bulk X₂PdY₆ (X=Nb, Ta; Y=S, Se) on the (010) plane. (a) Nb₂PdS₆, (b) Nb₂PdSe₆, (c) Ta₂PdS₆, and (d) Ta₂PdSe₆.

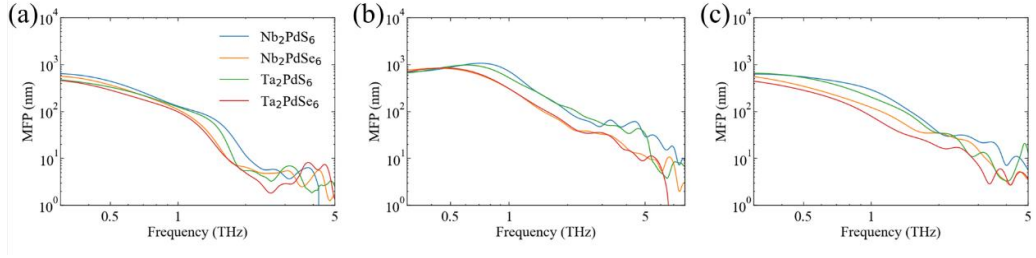


Figure S12. Phonon mean free path (MFP) of bulk X_2PdY_6 at 300K: (a) along the x -axis, (b) along the y -axis, and (c) along the z -axis.

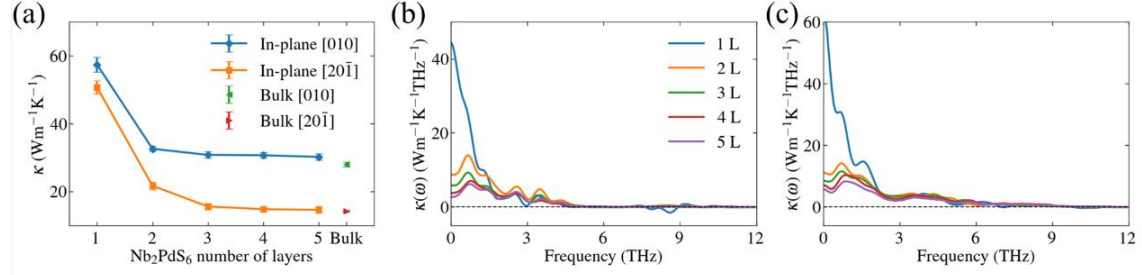


Figure S13. (a) The variation of Nb_2PdS_6 LTC with different layer numbers, with bulk LTC included for comparison; (b)-(c) Spectral decomposition of thermal conductivity along the in-plane $[20\bar{1}]$ and $[010]$ directions for 1-5 L.

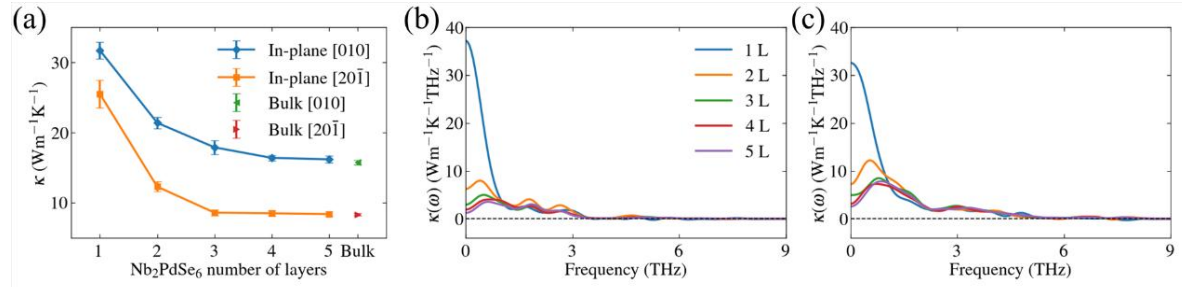


Figure S14. (a) The variation of Nb_2PdSe_6 LTC with different layer numbers, with bulk LTC included for comparison; (b)-(c) Spectral decomposition of thermal conductivity along the in-plane $[20\bar{1}]$ and $[010]$ directions for 1-5 L.

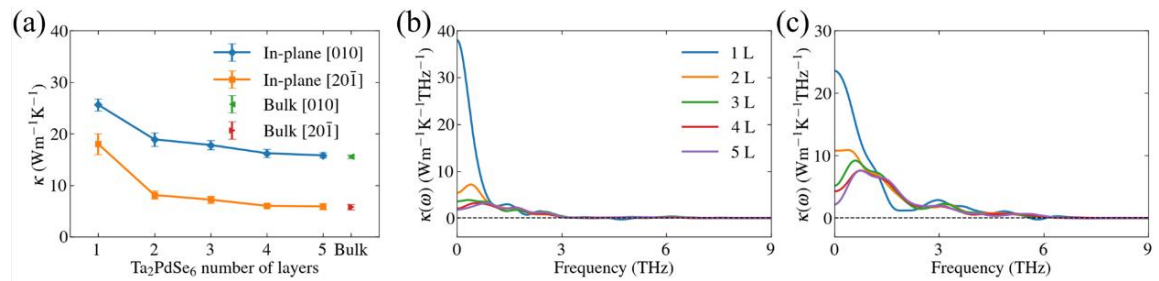


Figure S15. (a) The variation of Ta_2PdSe_6 LTC with different layer numbers, with bulk LTC included for comparison; (b)-(c) Spectral decomposition of thermal conductivity along the in-plane $[20\bar{1}]$ and $[010]$ directions for 1-5 L.

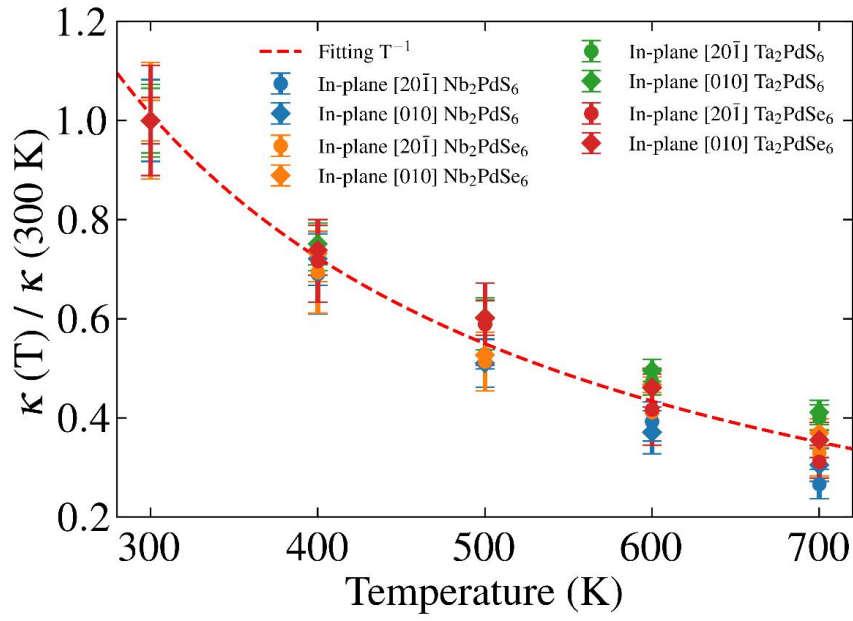


Figure S16. The reduced LTC of monolayer Nb_2PdS_6 , Nb_2PdSe_6 , Ta_2PdS_6 , and Ta_2PdSe_6 along the in-plane $[20\bar{1}]$ and $[010]$ crystal directions as a function of temperature. All LTC values are normalized by their respective values at 300 K.

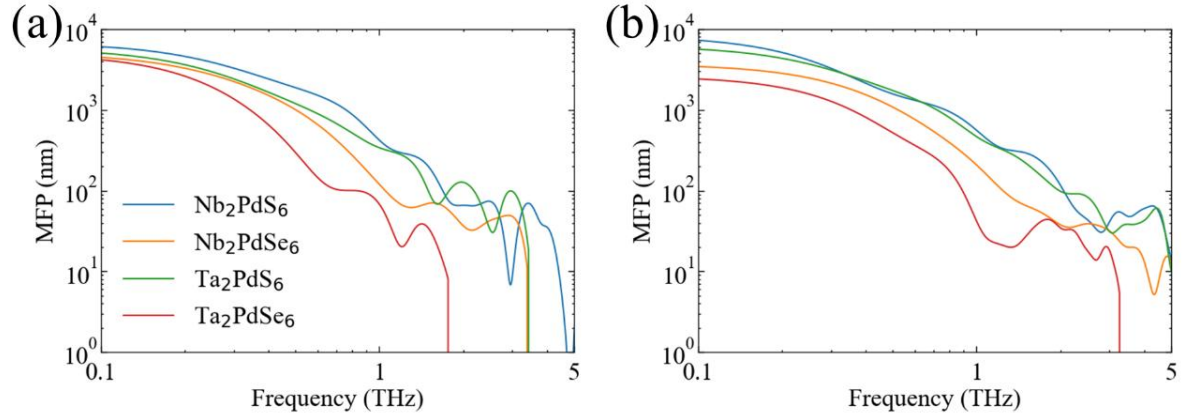


Figure S17. Phonon MFP of monolayer X_2PdY_6 at 300K: (a) along the x -axis and (b) along the y -axis.

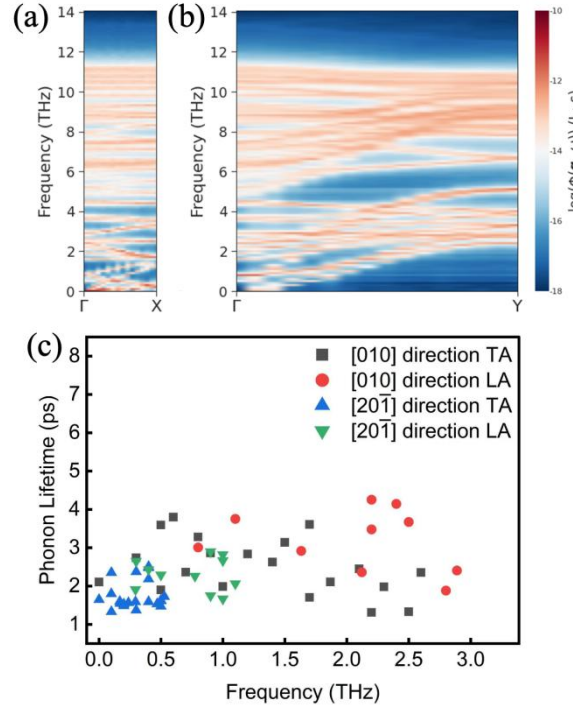


Figure S18. SED of monolayer Ta_2PdS_6 along the (a) $[20\bar{1}]$ direction and (b) $[010]$ direction. (c) phonon lifetimes of the resolved TA and LA modes along the $[010]$ and $[20\bar{1}]$ directions.

Because the 18-atom monolayer unit cell gives rise to 54 phonon branches, substantial overlap occurs among the SED peaks. Only clearly resolved LA and TA peaks were fitted using Lorentzian functions; therefore, the extracted lifetimes are used for qualitative directional comparison rather than exhaustive mode-by-mode quantification.

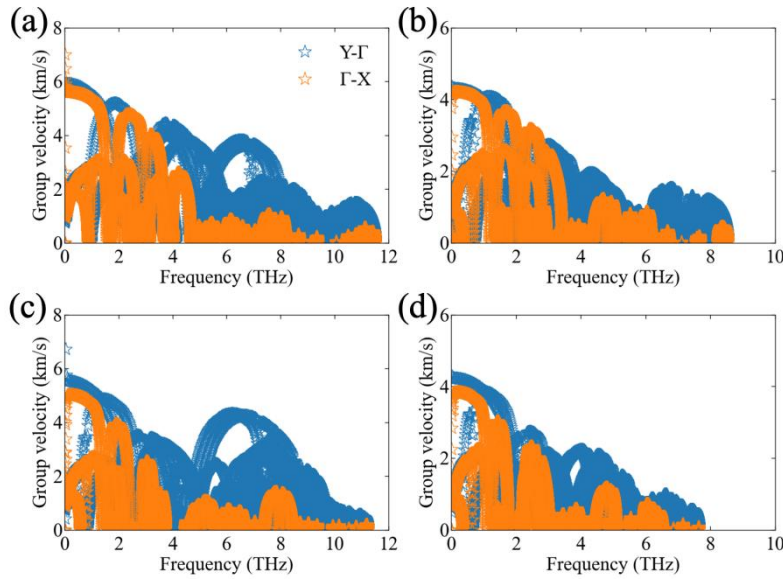


Figure S19. Phonon group velocity of monolayer structures along the $[20\bar{1}]$ direction (X) and $[010]$ direction (Y) calculated based on NEP for (a) Nb_2PdS_6 ; (b) Nb_2PdSe_6 ; (c) Ta_2PdS_6 ; (d) Ta_2PdSe_6 .

Table S1. The composition of the training dataset. All the four compounds use training datasets with the same composition.

Condition 1	Condition 2	Number of structures	Ratio
Bulk AIMD	100 K	10	2.5%
	300 K	10	2.5%
	500 K	10	2.5%
	800 K	10	2.5%
	1200 K	10	2.5%
	1600 K	10	2.5%
Bulk deformation + random displacement	$\pm 5\%$ and 0.1 Å	10	2.5%
	$\pm 5\%$ and 0.3 Å	10	2.5%
	$\pm 3\%$ and 0.1 Å	10	2.5%
	$\pm 3\%$ and 0.3 Å	10	2.5%
	$\pm 2\%$ and 0.1 Å	10	2.5%
	$\pm 2\%$ and 0.3 Å	10	2.5%
	$\pm 1\%$ and 0.1 Å	10	2.5%
	$\pm 1\%$ and 0.3 Å	10	2.5%
	$\pm 0.5\%$ and 0.1 Å	10	2.5%
	$\pm 0.5\%$ and 0.3 Å	10	2.5%
	0% and 0.1 Å	5	1.25%
	0% and 0.3 Å	5	1.25%
2D structure AIMD	100 K	20	5%
	300 K	20	5%
	600 K	20	5%
	1000 K	20	5%
2D structure deformation + random displacement	$\pm 2\%$ and 0.1 Å	10	2.5%
	$\pm 2\%$ and 0.3 Å	10	2.5%
	$\pm 1\%$ and 0.1 Å	10	2.5%
	$\pm 1\%$ and 0.3 Å	10	2.5%
	0% and 0.1 Å	5	1.25%
	0% and 0.3 Å	5	1.25%
Active learning		100	25%
Total		400	100%

Table S2. LTC tensor at 300 K.

$\kappa_{Nb_2PdS_6} = \begin{pmatrix} 2.98 & 0.0 & 1.80 \\ 0.0 & 27.99 & 0.0 \\ 1.80 & 0.0 & 13.99 \end{pmatrix}$	$\kappa_{Nb_2PdSe_6} = \begin{pmatrix} 2.06 & 0.0 & 0.90 \\ 0.0 & 15.75 & 0.0 \\ 0.90 & 0.0 & 8.22 \end{pmatrix}$
$\kappa_{Ta_2PdS_6} = \begin{pmatrix} 2.16 & 0.0 & 1.42 \\ 0.0 & 27.6 & 0.0 \\ 1.42 & 0.0 & 10.74 \end{pmatrix}$	$\kappa_{Ta_2PdSe_6} = \begin{pmatrix} 1.85 & 0.0 & 0.70 \\ 0.0 & 15.58 & 0.0 \\ 0.70 & 0.0 & 5.68 \end{pmatrix}$

Table S3. Comparison of κ_{xx} with $\kappa_{[102]}$ and of κ_{zz} with $\kappa_{[20\bar{1}]}$. LTC is reported in $Wm^{-1} K^{-1}$.

Materials	κ_{xx}	$\kappa_{[102]}$	Relative deviation (%)	κ_{zz}	$\kappa_{[20\bar{1}]}$	Relative deviation (%)
Nb ₂ PdS ₆	2.98	2.70	10.4%	13.99	14.27	2.0%
Nb ₂ PdSe ₆	2.06	1.94	6.2%	8.22	8.34	1.4%
Ta ₂ PdS ₆	2.16	1.93	11.9%	10.70	10.97	2.5%
Ta ₂ PdSe ₆	1.85	1.73	6.9%	5.69	5.80	1.9%