

Supplementary information

Loss-engineered scattering exceptional points in hyperbolic metacrystals for robust sensing

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Sec. A: Validity of phase compensation at the operating frequency

The thicknesses of the HMM and Si layers were chosen to give a total propagation phase of approximately π per period at the bandgap center frequency of 204.5 THz. As shown in Figure 2e of the main text, the opposite angular dispersions of the two layers keep the total phase nearly unchanged with incident angle. The localized states considered in this work, however, operate near 210.7 THz, away from the bandgap center. It is therefore necessary to examine whether the same compensation remains effective at the operating frequency.

We calculated the propagation phase accumulated over one period at 210.7 THz for different incident angles, with and without the intrinsic loss of the HMM. As shown in Figure S1, the total phase varies only slightly over the angular range considered. Its relative change from normal incidence to nearly 90° is 0.3367% when the intrinsic loss is neglected. With the loss included, the corresponding change in the real part of the total phase is 0.3319%. The two results are nearly identical. Although the operating frequency differs from the bandgap center frequency, the opposite angular phase variations of the HMM and Si layers still largely compensate each other, accounting for the weak angular dispersion of the localized resonances.

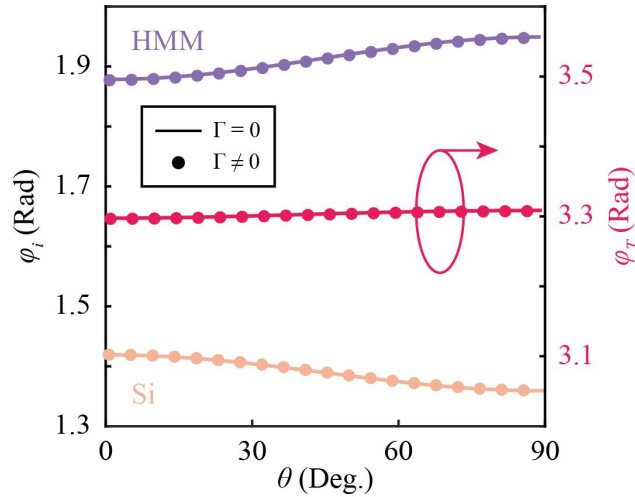


Figure S1. Angular dependence of the propagation phase accumulated in the HMM and Si layers and the total phase per period at 210.7 THz, calculated with and without the intrinsic loss of the HMM. The real part of the phase is plotted for the lossy case.

Sec. B: Analysis of scattering matrix exceptional point

Consider a two-port optical scattering system, whose schematic diagram is shown in Figure S2. The two ports of the system are port 1 (left) and port 2 (right). When electromagnetic waves are incident from both directions, the input and output fields of the system satisfy the following relations:

$$\begin{cases} E_2^- = tE_1^+ + r_2E_2^+ \\ E_1^- = r_1E_1^+ + tE_2^+ \end{cases} \quad (\text{B.1})$$

where E_1^+ and E_2^+ denote the electric field amplitudes incident from port 1 and port 2, respectively; E_1^- and E_2^- denote the electric field amplitudes outgoing from port 1 and port 2, respectively; r_1 and r_2 are the reflection coefficients for waves incident from port 1 and port 2, respectively; and t is the transmission coefficient. The physical meaning of these relations is as follows: the outgoing wave from port 2, E_2^- , consists of two components, one is the transmitted component tE_1^+ of the wave incident from port 1, and the other is the reflected component $r_2E_2^+$ of the wave incident from port 2. Similarly, the outgoing wave from port 1, E_1^- , is the superposition of the reflected component $r_1E_1^+$ of the wave incident from port 1 and the transmitted component tE_2^+ of the wave incident from port 2.

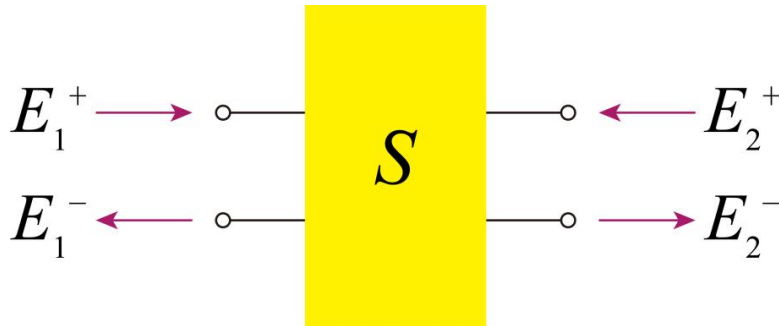


Figure S2. Schematic diagram of the two-port optical scattering system.

For the reciprocal two-port structure, we use the scattering-matrix (S -matrix) convention adopted in Ref. [48], with the outgoing fields ordered as those at ports 2 and 1, and scattering matrix $\mathbf{S}$ is given by

$$\mathbf{S} = \begin{pmatrix} t & r_2 \\ r_1 & t \end{pmatrix}. \quad (\text{B.2})$$

The two eigenvalues are $\lambda_{\pm} = t \pm \sqrt{r_1 r_2}$, giving an eigenvalue splitting of $\Delta\lambda = 2\sqrt{r_1 r_2}$. A necessary condition for their coalescence is $(r_1 r_2 = 0)$. To examine this condition in our structure, we calculated the complex reflection coefficients for forward and backward incidence independently using the transfer-matrix method (TMM). As shown in Figure S3, the real and imaginary parts of (r_1) simultaneously vanish at the marked operating point near 210.7 THz, while (r_2) remains finite. The system therefore satisfies the unidirectional reflectionless condition $(r_1 = 0)$, $(r_2 \neq 0)$. At this point, the S -matrix becomes

$$\mathbf{S}_{EP} = \begin{pmatrix} t_{EP} & r_2^{EP} \\ 0 & t_{EP} \end{pmatrix}. \quad (\text{B.3})$$

Both eigenvalues equal t_{EP} . Since $r_2^{EP} \neq 0$, the eigenvector equation has only one linearly independent solution, proportional to $(1, 0)^T$. The S -matrix is therefore non-diagonalizable, confirming a second-order exceptional point (EP). This result agrees with the unidirectional reflectionless scattering-EP condition demonstrated in Ref. [48].

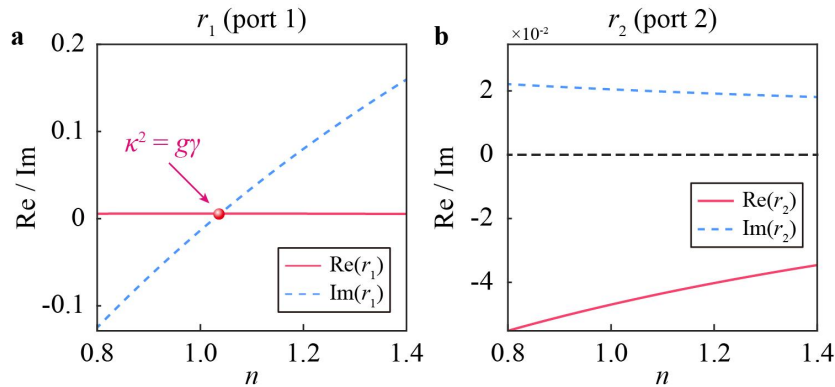


Figure S3. Directional reflection coefficients at 210.7 THz as a function of refractive index (n). Real and imaginary parts of the reflection coefficients for (a) forward

incidence (r_1) and (b) backward incidence (r_2). At the marked point, ($r_1 = 0$) while ($r_2 \neq 0$), corresponding to a unidirectional reflectionless S -matrix EP.

The same zero-reflection condition can be related to the effective Hamiltonian through the coupled-mode equations. At $\kappa^2 = g\gamma$, the forward reflection coefficient vanishes and the effective Hamiltonian has a real eigenvalue at (ω_0). These two results follow from the same driven scattering condition. The scattering EP additionally requires the backward reflection coefficient to remain nonzero, as verified in Figure S3. It is worth distinguishing this reflection zero from a natural resonance pole. The former is a zero of the driven reflection amplitude, whereas the latter is determined by the complex eigenfrequency of the undriven open system. Likewise, the real eigenvalue of the zero-reflection effective Hamiltonian describes the driven scattering solution and does not, by itself, establish zero radiative decay of a passive eigenmode.

For the coupled-mode model considered here, the reflection coefficient can be expressed as [S1]

$$r_1(\omega) = -\frac{\det(\omega\mathbf{I} - \mathbf{H}_e)}{\det(\omega\mathbf{I} - \mathbf{H}_o)}. \quad (\text{B.4})$$

where $\mathbf{H}_e$ is the effective Hamiltonian obtained under the zero-reflection boundary condition, while $\mathbf{H}_o$ describes the natural modes of the undriven structure. The zeros of the numerator determine the reflection-zero solutions, whereas the zeros of the denominator determine the resonance poles, apart from possible pole-zero cancellations. The two Hamiltonians differ in the sign of the radiative contribution associated with port 1. This difference arises from the input–output boundary condition imposed on the driven system. In particular, a real eigenvalue of $\mathbf{H}_e$ corresponds to a zero of the reflection amplitude at a real frequency, but does not imply that the corresponding undriven mode has zero radiative decay. The S -matrix EP is determined separately from the directional reflection coefficients, as discussed above.

When the refractive index at the incident port n undergoes a small perturbation Δn , the elements of the S -matrix vary accordingly. Performing a first-order Taylor expansion of the S -matrix at the EP:

$$S = S^{\text{EP}} + \frac{\partial S}{\partial n} \Delta n = \begin{pmatrix} t^{\text{EP}} + \frac{\partial t}{\partial n} \Delta n & r_2^{\text{EP}} + \frac{\partial r_2}{\partial n} \Delta n \\ \frac{\partial r_1}{\partial n} \Delta n & t^{\text{EP}} + \frac{\partial t}{\partial n} \Delta n \end{pmatrix}. \quad (\text{B.5})$$

Substituting this into the expression for the eigenvalues of the S -matrix, the perturbed eigenvalue splitting is obtained as:

$$\Delta v = 2 \sqrt{\frac{\partial r_1}{\partial n} \Delta n \left(r_2^{\text{EP}} + \frac{\partial r_2}{\partial n} \Delta n \right)}. \quad (\text{B.6})$$

Since Δn is a small perturbation, the second-order term in Δn can be neglected, and the derivative of r_1 with respect to the refractive index at the incident port remains approximately constant. Thus, the above expression can be approximated as:

$$\Delta v \propto \sqrt{\Delta n}. \quad (\text{B.7})$$

Sec. C: Derivation of coupled-mode theory

This supplementary material provides a detailed derivation of the coupled-mode theory used in the main text. In the case of single-port excitation, a single resonant mode can be treated as a resonance mode $\tilde{a}_0 = a_0 e^{-i\omega t}$, and the dynamical equation based on the Lorentz model can be written as:

$$\frac{d\tilde{a}_0}{dt} = (-i\omega_0 - \gamma_{01} - \gamma_{02} - \Gamma) \tilde{a}_0 + \sqrt{2\gamma_{01}} S_{1+} e^{-i\omega t}, \quad (\text{C.1})$$

where ω_0 is the resonance frequency, Γ is the intrinsic loss, and γ_{01} and γ_{02} denote the scattering losses of the mode at the input port (port 1) and the output port (port 2), respectively. The reflection coefficient at port 1 is given by:

$$r = -1 + \frac{\sqrt{2\gamma_{01}} a_0}{S_{1+}} = -1 + \frac{2i\gamma_{01}}{(\omega - \omega_0) + i(\gamma_{01} + \gamma_{02} + \Gamma)}. \quad (\text{C.2})$$

From the above equation, the reflectance at resonance $R(\omega_0)$ and the linewidth $X = \gamma_{01} + \gamma_{02} + \Gamma$ satisfy:

$$R(\omega_0) = \left(-1 + \frac{2\gamma_{01}}{X} \right)^2. \quad (\text{C.3})$$

Solving this gives:

$$\gamma_{01} = \frac{X}{2} \left(1 \pm \sqrt{R(\omega_0)} \right). \quad (\text{C.4})$$

The sign in Eq. (C.4) depends on the relative magnitudes of the radiative and intrinsic losses. From Eq. (C.3), the reflectance at resonance can be written as

$$R(\omega_0) = \left(\frac{\gamma_{01} - \gamma_{02} - \Gamma}{X} \right)^2, \quad (\text{C.5})$$

where $X = \gamma_{01} + \gamma_{02} + \Gamma$. The positive sign in Eq. (C.4) corresponds to $\gamma_{01} > \gamma_{02} + \Gamma$, whereas the negative sign corresponds to $\gamma_{01} < \gamma_{02} + \Gamma$. In the absence of intrinsic loss, the condition reduces to $\gamma_{01} > \gamma_{02}$ or $\gamma_{01} < \gamma_{02}$, respectively.

To determine the appropriate branch, we vary the number of periods on one side of the isolated structure, which changes the radiative loss at the corresponding port. The reflectance reaches zero when $\gamma_{01} = \gamma_{02} + \Gamma$. The variation of the reflectance around this condition, together with the known change in radiative leakage, allows the two regimes to be distinguished. When Γ is nonzero, the reflection spectrum determines γ_{01} and the combined loss $\gamma_{02} + \Gamma$. The reflection spectrum provides the resonant reflectance and linewidth, from which the radiative loss at the incident port and the combined loss from the opposite port and intrinsic absorption can be determined. These two contributions can be separated using the transmittance at resonance, and the transmittance at resonance is given by

$$T(\omega_0) = \frac{4\gamma_{01}\gamma_{02}}{X^2}. \quad (\text{C.6})$$

With γ_{01} and X obtained from the reflection spectrum, γ_{02} can be calculated from the transmittance, and Γ then follows from $X = \gamma_{01} + \gamma_{02} + \Gamma$. These extracted parameters are subsequently used in the coupled-mode model. This procedure is applied separately to the isolated defect and interface modes to obtain the loss parameters used in the

coupled-mode model. In the two-mode coupled model, the resonance frequencies and loss rates are extracted from the corresponding isolated structures. This avoids the mixing of the two resonances in the coupled system and allows the parameters of each mode to be determined separately. Since both modes are spatially localized, their resonance frequencies and intrinsic losses are mainly influenced by the nearby layers. The radiative loss is primarily determined by the number of periods between each mode and its adjacent external port. When sufficient periods are present, changes in the more distant layers have a relatively small effect on these parameters.

The isolated-mode parameters are therefore used as approximate uncoupled values in the CMT model, while the interaction between the two states is described by the coupling strength κ , which varies with the intermediate period number q . Some deviations may occur at smaller separations due to stronger modal overlap. The agreement between the TMM and CMT results in the main text supports the applicability of this approximation over the parameter range considered.

For a system consisting of two coupled modes, let the complex amplitudes of the two modes be $\tilde{a}_1 = a_1 e^{-i\omega t}$ and $\tilde{a}_2 = a_2 e^{-i\omega t}$. The dynamical equations can be written as:

$$\begin{aligned}\frac{d\tilde{a}_1}{dt} &= (-i\omega_0 - \gamma_1 - \Gamma_1)\tilde{a}_1 - i\kappa\tilde{a}_2 + \sqrt{2\gamma_1}S_{1+}e^{-i\omega t}, \\ \frac{d\tilde{a}_2}{dt} &= (-i\omega_0 - \gamma_2 - \Gamma_2)\tilde{a}_2 - i\kappa\tilde{a}_1.\end{aligned}\tag{C.7}$$

Here $\sqrt{2\gamma_1}S_{1+}$ is the external excitation, γ_1 and γ_2 are the radiative losses describing the energy leakage of the two modes into the external radiation channels, Γ_1 and Γ_2 are the intrinsic losses describing the non-radiative dissipation inside the system, and κ is the coupling strength between the two modes. The reflection coefficient at port 1 is given by:

$$r = -1 + \frac{\sqrt{2\gamma_1}a_1}{S_{1+}}.\tag{C.8}$$

Under the zero-reflection condition at port 1,

$$S_{1-} = -S_{1+} + \sqrt{2\gamma_1}a_1 = 0,\tag{C.9}$$

the external excitation can be expressed in terms of the mode amplitude as

$$\sqrt{2\gamma_1}S_{1+} = 2\gamma_1 a_1. \quad (\text{C.10})$$

Substituting this relation into the first equation of Eq. (C.7), the dynamical equation for the first mode becomes

$$\frac{d\tilde{a}_1}{dt} = (-i\omega_0 + \gamma_1 - \Gamma_1)\tilde{a}_1 - i\kappa\tilde{a}_2. \quad (\text{C.11})$$

Together with the second equation of Eq. (C.7), the coupled dynamics under the zero-reflection condition can be written in matrix form as

$$i\frac{d}{dt}\begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{pmatrix} = \begin{pmatrix} \omega_0 + i(\gamma_1 - \Gamma_1) & \kappa \\ \kappa & \omega_0 - i(\gamma_2 + \Gamma_2) \end{pmatrix} \begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{pmatrix}, \quad (\text{C.12})$$

which yields the effective Hamiltonian

$$H_e = \begin{pmatrix} \omega_0 + i(\gamma_1 - \Gamma_1) & \kappa \\ \kappa & \omega_0 - i(\gamma_2 + \Gamma_2) \end{pmatrix}. \quad (\text{C.13})$$

Therefore, the positive imaginary term originates from the incident field and represents effective gain in the corresponding eigenvalue problem, rather than gain supplied by the material. A similar treatment has been used to construct effective non-Hermitian Hamiltonians in passive coupled-resonator systems [S2, S3]. The physical system remains open, with both radiative leakage and intrinsic dissipation. The effective Hamiltonian therefore differs from the Hamiltonian describing the natural modes of the undriven structure.

The relation between the zero-reflection condition and the real eigenvalue of the effective Hamiltonian can be obtained directly from the coupled-mode equations. At the common resonance frequency (ω_0), the forward reflection coefficient is

$$r_1(\omega_0) = \frac{(\gamma_1 - \Gamma_1)(\gamma_2 + \Gamma_2) - \kappa^2}{(\gamma_1 + \Gamma_1)(\gamma_2 + \Gamma_2) + \kappa^2}, \quad (\text{C.14})$$

The reflection vanishes when $\kappa^2 = (\gamma_1 - \Gamma_1)(\gamma_2 + \Gamma_2)$. Under the same condition, the effective Hamiltonian in Eq. (C.13) has a real eigenvalue ($\omega_1 = \omega_0$), while the other eigenvalue is $\omega_2 = \omega_0 + i(g - \gamma)$. Thus, the zero-reflection condition and the real-frequency eigenvalue follow from the same coupled-mode relation. This correspondence is consistent with previous analyses of driven non-Hermitian resonator systems. The positive imaginary term in the effective Hamiltonian arises from the

imposed zero-reflection boundary condition and represents effective gain from the external excitation. The real eigenvalue therefore describes the corresponding driven scattering solution, rather than directly giving the natural resonance pole of the undriven optical structure.

Sec. D: Reflectance spectra under refractive-index perturbations

The sensing response in the main text is characterized by both the resonance-frequency shift and the splitting of the S -matrix eigenvalues. To show how the refractive-index perturbation affects a directly measurable optical spectrum, Figure S4 presents the reflectance spectra of the coupled structure under normal incidence for different values of Δn . As the refractive index increases, the resonance dip shifts gradually while remaining well defined. The corresponding frequency shifts are extracted from these spectra and plotted in Figure 7d of the main text. Their small magnitude and approximately linear dependence on Δn indicate that the resonance frequency remains relatively stable under the perturbations considered.

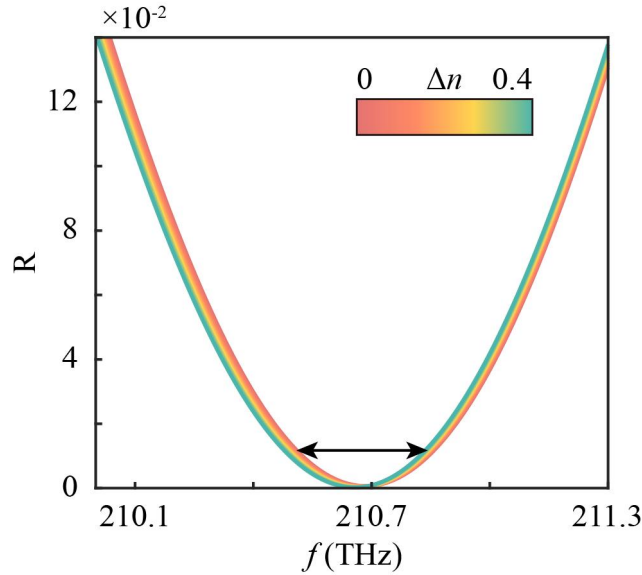


Figure S4. Reflectance spectra of the coupled structure under normal incidence for different refractive-index perturbations Δn . The resonance-frequency shifts extracted from these spectra are plotted in Figure 7d of the main text.

The scattering eigenvalue splitting describes a different response. Rather than appearing as a pronounced movement of the resonance dip, the square-root dependence arises from changes in the directional reflection coefficients. For the reciprocal two-port S -matrix considered here, the magnitude of the eigenvalue splitting can be reconstructed from the forward and backward reflectance measured at the same frequency. The full complex eigenvalues, however, require phase-resolved scattering information. Thus, the spectra in Figure S4 provide a conventional optical view of the resonance-frequency response, while the enhanced scattering response is characterized separately through the S -matrix analysis.

Sec. E: Angular robustness and sensing response under structural perturbations

In the fabricated HMM, variations in the thicknesses of the ITO/Si sublayers may change the effective permittivity and weaken the phase compensation. To examine this effect, we introduced random variations in the HMM filling ratio across the ten periods of PC1 for the structure with ($q = 7$). The filling-ratio deviations were limited to ± 0.1 , corresponding to HMM thickness variations of ± 30 nm. The total thickness of each period was kept unchanged, so the thicknesses of the adjacent dielectric layers varied accordingly. The selected thickness variations are shown in Figure S5a. Despite these deviations, the full-angle reflectance spectra in Figure S5b retain a nearly angle-independent resonance near the original operating frequency. This indicates that the angular phase compensation is not strongly affected by the structural deviations considered here.

We further examined the reflection spectra and scattering eigenvalue splitting of the perturbed structure at different incident angles. As shown in Figure S6a, the reflection minima remain close in frequency over the range of 0° - 60° . The corresponding scattering eigenvalue splitting in Figure S6b exhibits an approximately square-root dependence on the refractive-index perturbation, with the fitted slopes in Figure S6c

remaining close to 0.5, even at 60° . These results show that the angular stability and characteristic scattering response survive the structural deviations examined here. The calculation represents one random realization of the thickness errors and does not establish a statistical fabrication yield. Moreover, the results describe the noise-free scattering response. Although the eigenvalue splitting increases rapidly near the operating point, the noise-equivalent refractive-index resolution depends on the uncertainties in the measured scattering coefficients and cannot be determined from the present calculations alone.

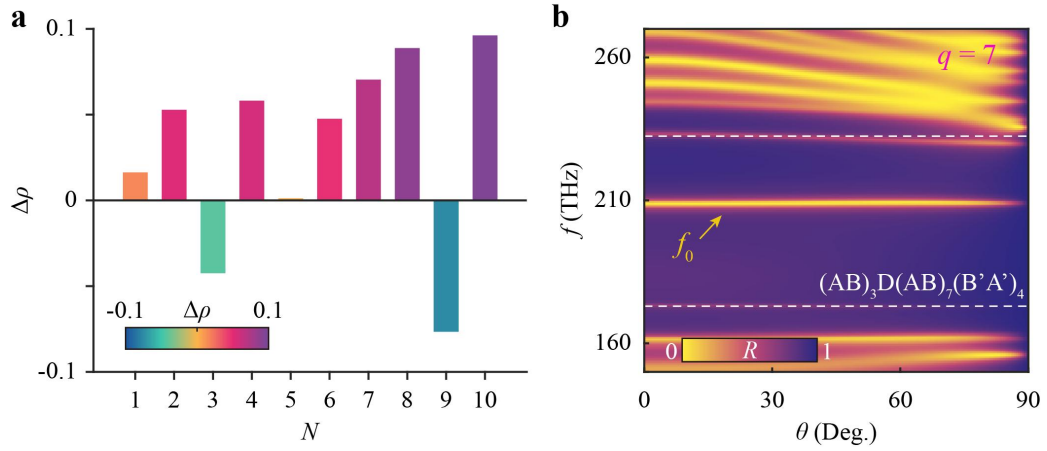


Figure S5. Angular robustness under structural perturbations. (a) Random variations in the HMM filling ratio across the ten periods of PC1. (b) Angle-resolved reflectance spectra of the perturbed structure.

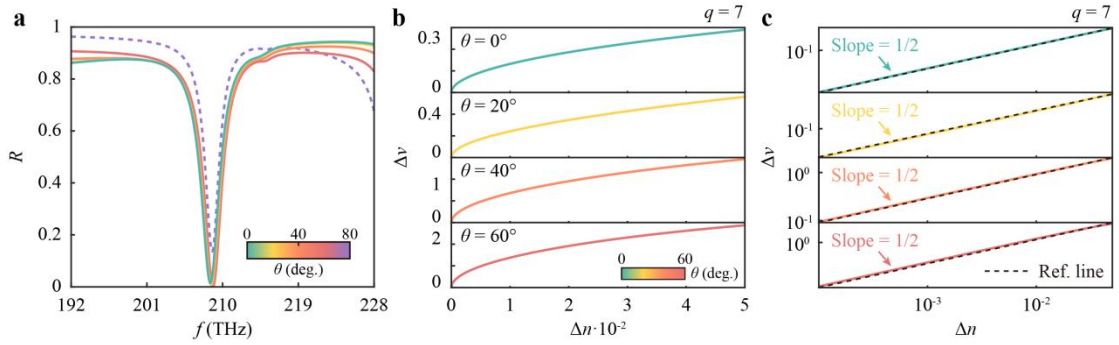


Figure S6. Reflection and scattering responses of the perturbed structure. (a) Reflectance spectra at incident angles of 0° , 20° , 40° , 60° , and 80° . (b) Scattering eigenvalue splitting as a function of the refractive-index perturbation (Δn) at different incident angles. (c) Corresponding log-log plots and linear fits.

Sec. F: Comparison with Hamiltonian-EP, BIC-related, and scattering-EP sensing

To examine the roles of hyperbolic dispersion and loss engineering in the proposed sensing scheme, we consider an isotropic photonic-crystal (PC) heterostructure as a reference. By adjusting its coupling and loss parameters, three operating conditions are examined: a Hamiltonian EP, a BIC-related real-frequency state, and a S -matrix EP. The corresponding refractive-index responses, resonance-frequency shifts, and angular dependences are calculated using the same TMM and coupled-mode theory (CMT) employed in the main text. The reference structure consists of a one-dimensional PC heterostructure, $(A'B'A')_{N_1}(C'D'C')_q(A'B'A')_{N_2}$, where A' and D' are SiC ($\epsilon_{\text{SiC}} = 6.6$), and B' and C' are Si. The three sections, $(A'B'A')_{N_1}$, $(C'D'C')_q$, and $(A'B'A')_{N_2}$ are denoted as PC_I, PC_{II}, and PC_{III}, respectively. Each unit cell contains Si and SiC layers with total thicknesses of 150 nm and 105.3 nm, respectively. This arrangement supports localized interface states at the PC_I–PC_{II} and PC_{II}–PC_{III} boundaries. The period numbers N_1 and N_2 determine the radiative coupling to the external ports, while q controls the interaction between the two interface states. When $N_1 = N_2$, a PT -symmetric effective Hamiltonian can be established, and the Hamiltonian EP is reached by varying q . For $N_1 \neq N_2$, the effective gain and loss become unbalanced, allowing a real-frequency solution to appear in the weak-coupling regime. The scattering response at this operating point is then examined through the S -matrix.

F.1 Comparison with Hamiltonian-EP sensing

We first consider the PT -symmetric case with $N_1 = N_2$. The coupling strength extracted for different intermediate period numbers is plotted in Figure S7a. The resonance frequencies obtained from the TMM spectra agree with the eigenvalues of the effective Hamiltonian, as shown in Figure S7b, with the Hamiltonian EP occurring at $q = 12$. The sensing response at this point is examined by varying the refractive index at the incident port. Under normal incidence, the resonance-frequency shift follows a square-root dependence on the perturbation, with a fitted log–log slope close to 0.5 [Figure S7c].

This behavior changes when the incident angle deviates from normal incidence. At 5° , the fitted response becomes approximately linear, as shown in Figure S7d. The angular variation shifts the system away from its original EP condition, modifying the response to refractive-index perturbations. The comparison shows that, for this reference structure, the Hamiltonian-EP response is sensitive to even a small change in the incident angle.

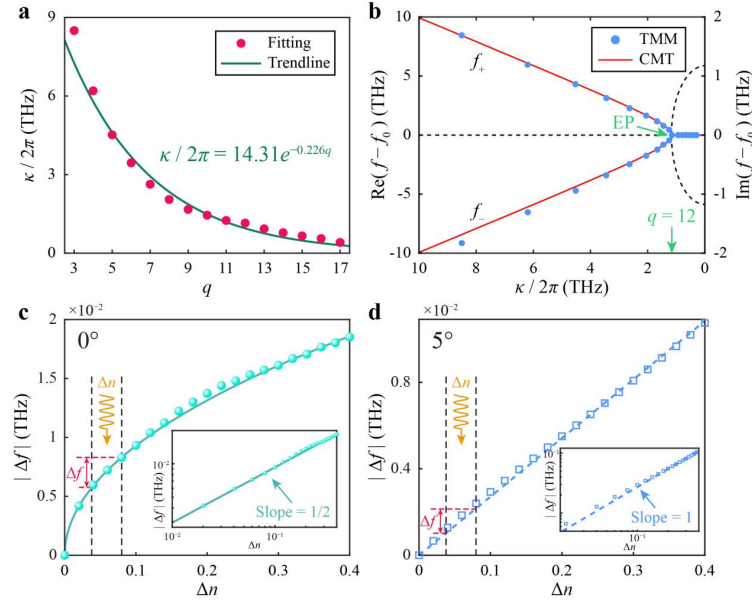


Figure S7. Hamiltonian-EP sensing in the PT -symmetric reference structure. (a) Coupling strength as a function of the intermediate period number q . (b) Eigenvalues of the effective Hamiltonian and resonance frequencies extracted from the TMM spectra, with the EP occurring at $q = 12$. (c) Resonance-frequency shift versus refractive-index perturbation under normal incidence, with a fitted log–log slope close to 0.5. (d) Corresponding results at an incident angle of 5° , showing an approximately linear dependence.

F.2 Comparison with BIC-related sensing

We next consider an asymmetric configuration with $N_1 = 5$ and $N_2 = 9$. The variation of the coupling strength with q is plotted in Figure S8a, while the corresponding eigenvalues of the effective Hamiltonian are compared with the TMM resonance frequencies in Figure S8b. Changing N_2 has little effect on the coupling strength but

alters the effective gain–loss balance. At $q = 14$, one eigenvalue becomes purely real in the weak-coupling regime, giving the BIC-related real-frequency condition discussed in the main text. The resonance-frequency shift at this point is approximately linear with the refractive-index perturbation, as shown in Figure S8c. Compared with the Hamiltonian EP, the frequency response no longer exhibits the square-root dependence. The angular behavior is examined in Figure S8d, where the reflection resonance shifts progressively toward higher frequencies as the incident angle increases. Such angular dispersion would cause the resonance to move away from a fixed operating frequency. In contrast, the HMM-integrated structure in Figure 6e of the main text exhibits much smaller angular shifts, with the resonance remaining nearly unchanged over the investigated angular range.

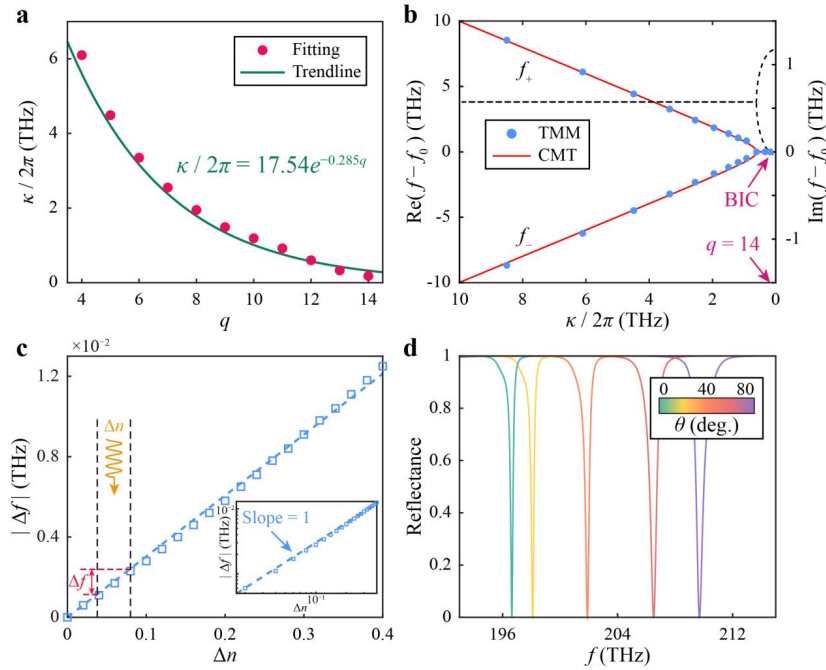


Figure S8. BIC-related sensing in the PT -asymmetric reference structure. (a) Coupling strength versus the intermediate period number q . (b) Effective Hamiltonian eigenvalues and resonance frequencies obtained from the TMM spectra, showing a real-frequency solution at $q = 14$. (c) Resonance-frequency shift versus refractive-index perturbation at this operating point, exhibiting an approximately linear dependence. (d) Angle-resolved reflectance spectra showing a progressive blueshift with increasing incident angle.

F.3 Comparison with scattering-EP sensors

Although the BIC-related operating point exhibits a linear resonance-frequency response, a square-root dependence can be recovered by examining the scattering eigenvalues instead. At normal incidence, the S -matrix eigenvalue splitting follows a square-root dependence on the refractive-index perturbation, as shown in Figure S9a. The corresponding log–log fit gives a slope close to 0.5. The angular stability of this response depends on whether the scattering EP condition remains satisfied. For the two-port system considered here, the eigenvalue splitting is determined by $\Delta\nu = 2\sqrt{r_1 r_2}$, so that the EP requires one directional reflection coefficient to vanish while the other remains finite. As shown in Figure S9b, the product $r_1 r_2$ departs from zero as the incident angle increases, indicating that the system moves away from the scattering EP. Thus, although the square-root response can be obtained under normal incidence, it is not preserved over a broad angular range in this isotropic reference structure.

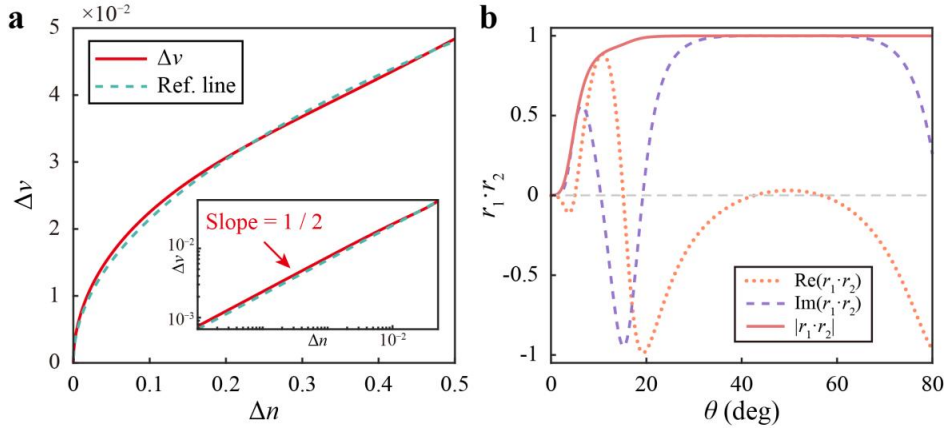


Figure S9. Scattering-EP response of the reference structure. (a) S -matrix eigenvalue splitting versus refractive-index perturbation under normal incidence. The inset shows the corresponding log–log fit with a slope close to 0.5. (b) Angular dependence of $r_1 r_2$, showing its departure from zero as the incident angle increases.

The three reference cases illustrate how the sensing response depends on the operating condition. The Hamiltonian EP provides a square-root resonance-frequency response under normal incidence, but the response changes markedly with a small angular deviation. The BIC-related real-frequency state exhibits an approximately

linear frequency response, while its resonance remains sensitive to the incident angle. Scattering-EP sensing restores the square-root eigenvalue splitting at normal incidence, although the EP condition is lost as the angle increases. In the HMM-integrated structure, the opposite angular dispersions of the HMM and dielectric layers largely compensate each other, keeping the localized resonances nearly unchanged with incident angle. The intrinsic loss provides an additional means of adjusting the non-Hermitian operating point, allowing a scattering EP to be realized away from the conventional Hamiltonian EP. As shown in Figure 7 of the main text, the resonance-frequency shift is reduced, while the scattering eigenvalue splitting retains a fitted exponent close to 0.5 over the investigated angular range. The comparison indicates that the advantage of the proposed structure comes from combining angular phase compensation with loss control, allowing a pronounced scattering response to be maintained with reduced resonance-frequency drift.

Supplementary references

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