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Supplementary information

Spin-selective Fourier transformation for parity-resolved optical vortex manipulation

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Note 1. Derivation of the parity phase induced by a 4f imaging system

Consider an input OAM beam with a complex amplitude in the initial plane (x, y):

$$U_{in}(x, y) = A(r) \cdot \exp(il\theta), \tag{S1}$$

where $A(r)$ is the radial amplitude distribution, l is the topological charge, and θ is the azimuthal angle. In Cartesian coordinates, a point is expressed as (x, y). The corresponding polar coordinates (r, θ) satisfy:

$$x = r\cos\theta,\ y = r\sin\theta, \tag{S2}$$

where $r = \sqrt{x^2 + y^2}$ and $\theta = \arctan(y/x)$.

In an ideal 4f configuration, the field at the output plane is a scaled, inverted replica of the input:

$$U_{out}(x', y') \propto F\{F\{U_{in}(x, y)\}\} = U_{in}(-x, -y), \tag{S3}$$

where F represent the Fourier transformation. The coordinate inversion maps a point as:

$$(x, y) \to (-x, -y). \tag{S4}$$

Substituting the polar representation:

$$\begin{aligned} -x &= -r\cos\theta = r\cos(\theta + \pi), \\ -y &= -r\sin\theta = r\sin(\theta + \pi). \end{aligned} \tag{S5}$$

Comparing the transformed expressions with the polar coordinate definition:

$$(-x, -y) = (r\cos(\theta + \pi), r\sin(\theta + \pi)). \tag{S6}$$

Thus, the polar coordinates of the transformed point are:

$$\begin{aligned} r' &= \sqrt{(-x)^2 + (-y)^2} = r, \\ \theta' &= \arctan\left(\frac{-y}{-x}\right) = \theta + \pi. \end{aligned} \tag{S7}$$

The polar coordinate form of the output OAM field is:

$$\begin{aligned} U_{out}(r', \theta') &\propto U_{in}(r, \theta + \pi) \\ &= A(r) \cdot \exp[il(\theta + \pi)] \\ &= [A(r)\exp(il\theta)] \cdot \exp(il\pi) \\ &= U_{in}(r, \theta) \cdot \exp(il\pi). \end{aligned} \tag{S8}$$

Note 2. Experimental setup for OAM mode separation

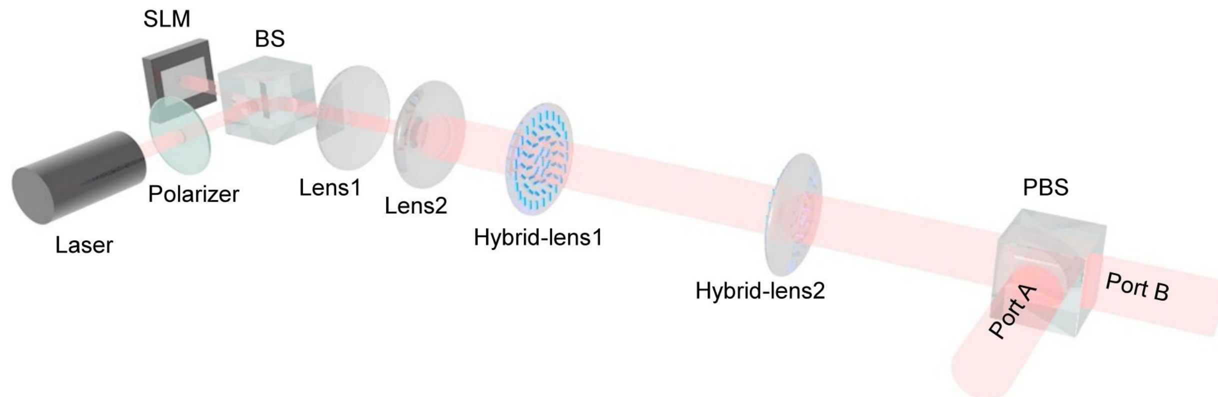

Figure S1. Experimental setup for OAM mode separation. SLM: spatial light modulator; BS: beam splitter; PBS: polarization-dependent beam splitter.

Note 3. Influence of the spot-size mismatch on OAM mode separation

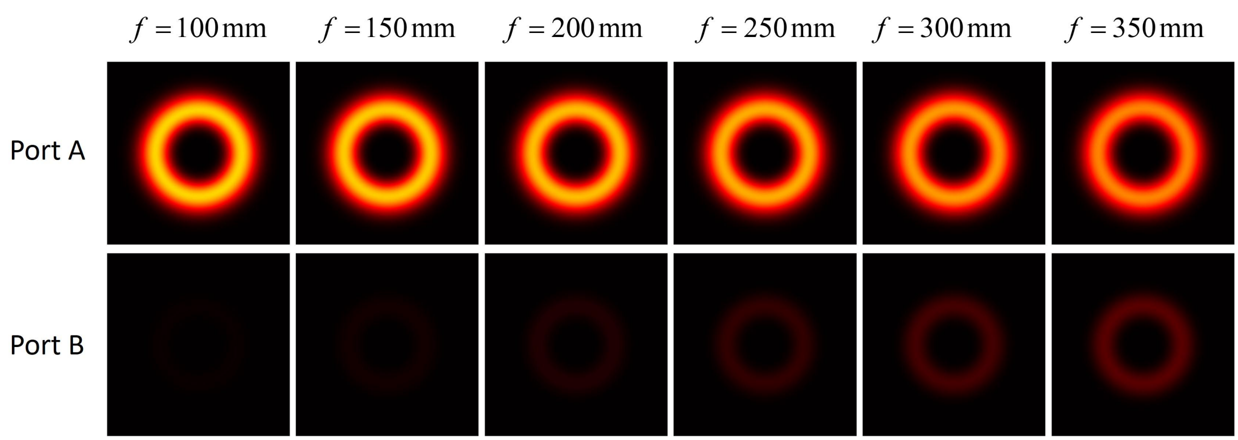

Figure S2. Simulated output intensity distributions at Port A and Port B for different focal lengths of the PB lens, assuming an input even mode ($l = +4$).

Note 4. Quantitative performance metrics

We measured the input power P_{in} (before the first hybrid lens) and the output powers P_A and P_B (at the two PBS ports) for each tested OAM mode using a calibrated power meter. The results are summarized in Table S1.

Table S1. Quantitative performance metrics

Input mode	Parity	Separation Efficiency	Insertion Loss (dB)	Extinction Ratio (dB)	Polarization Fidelity	Crosstalk (dB)
+1	odd	90.3%	0.44	18.25	98.5%	-18.25
-1	odd	90.2%	0.45	15.58	97.3%	-15.58
+2	even	91.0%	0.41	11.23	93.0%	-11.23
-2	even	91.6%	0.38	14.57	96.6%	-14.57
+3	odd	91.8%	0.37	11.34	93.2%	-11.34
-3	odd	90.2%	0.45	11.57	93.5%	-11.57

Definition of metrics:

- Separation efficiency: $\eta = \left(P_A + P_B\right)/P_{in} \times 100\%$
- Insertion loss: $IL = -10\log_{10}\left(\eta\right)\ dB$
- Extinction ratio (even): $ER_{even} = 10\log_{10}\left(P_A/P_B\right)\ dB$
- Extinction ratio (odd): $ER_{odd} = 10\log_{10}\left(P_B/P_A\right)\ dB$
- Polarization fidelity: $F = P_{target}\big/\left(P_{target} + P_{orthogonal}\right)$ for the expected output polarization
- Crosstalk: Crosstalk $\approx -ER$ dB

The separation efficiency exceeds 90% for all tested modes, with an average value of 90.9%, corresponding to an average insertion loss of 0.42 dB. The extinction ratio ranges from 11.23 dB to 18.25 dB, with an average of 13.76 dB. This corresponds to an average polarization fidelity of 95.3%. The crosstalk, defined as the power fraction leaking to the wrong port, is below -11 dB for all tested modes.

The variation in extinction ratio across different modes is primarily attributed to the slight deviation of the PB lens from the ideal half-wave retardation and residual optical misalignments.

The asymmetry between positive and negative topological charges (e.g., +2 vs. -2) is likely due to minor aberrations in the optical system or slight differences in the SLM-generated mode purity. We note that the extinction ratio of 11 dB, although the lowest among the tested modes, still corresponds to a polarization fidelity of ~93%, which is sufficient for effective parity separation.

Note 5. Influence of the phase retardance deviation on OAM mode separation

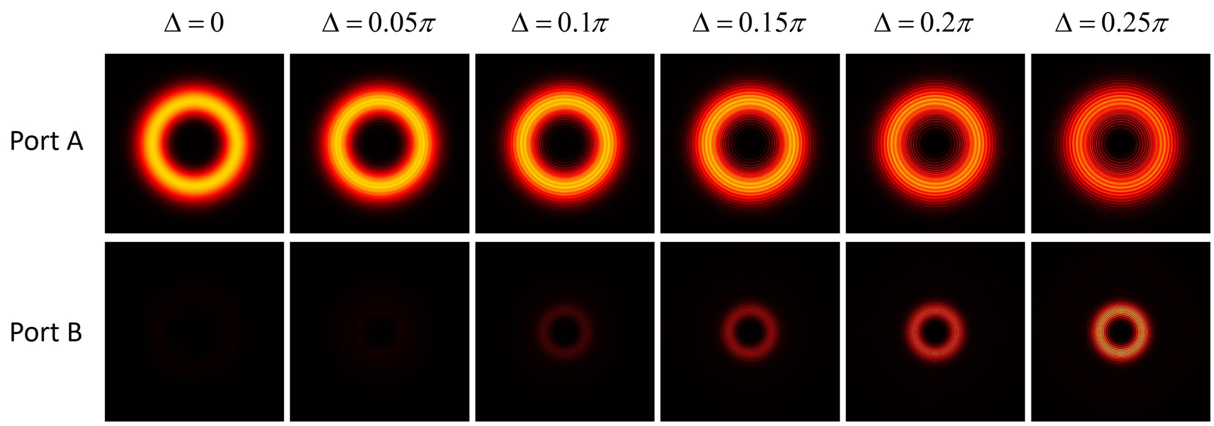

Figure S3. Simulated output intensity distributions at Port A and Port B for different phase retardance deviations of the PB lens, assuming an input even mode (l = +4).