



A cumulative model for thermoelectric cooling with temperature dependent material properties

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Abstract

The precise evaluation of real-world thermoelectric cooling (TEC) performance is vital for both the development of TEC devices and the rational design of TEC materials. Due to the relatively limited temperature difference (ΔT) that a single-stage TEC can achieve, the classical analytical formulae for thermoelectric cooling are commonly used with averaged material properties in the cooling performance evaluation. Nevertheless, this could cause remarkable errors in evaluations of the cooling power, even under a ΔT as low as 10 K. In this work, a cumulative model for thermoelectric cooling is proposed to evaluate the maximum coefficient of performance (COP) as well as the maximum cooling power of a TEC device under finite temperature differences, with the temperature dependence of material properties being fully considered. The inherent deficiencies in predicting the maximum cooling power by both the classical formulae and the original cumulative model are identified, and effective refinements are implemented to reduce the prediction error. Eight thermoelectric materials with state-of-the-art ZT values near room temperature are assessed for electronics cooling scenarios within a cold side temperature (T_c) range of 300-350 K. In comparison to the classical formulae, the proposed model predicts the maximum COP with comparably satisfying accuracies ($\pm 2\%$), while significantly improves the prediction accuracy of the maximum cooling power from approximately $\pm 30\%$ to within $\pm 5\%$ over a ΔT range of 10-70 K. This work fills the gap between thermoelectric material properties and device-level cooling performance, and is beneficial to the development and application of thermoelectric conversion technology for cooling purposes.

Keywords: Thermoelectric cooling, performance evaluation, cumulative model, temperature dependence, material properties

1. Introduction

A thermoelectric module can directly convert heat into electric power (the power generation mode) under a temperature gradient and enable heat flow from lower temperature to higher temperature (the cooling or heat pump mode) at the cost of electric power input. The power generation mode is promising for solar energy utilization^[1], waste heat recovery^[2] and human body energy harvesting^[3], while the cooling mode is of great use for the active thermal management of high-power density electronics^[4] and precise temperature control of critical components in optoelectronic devices^[5] as well as spacecraft systems^[6]. To make full use of this technology, the precise evaluation of its real-world performance is essential.

In 1957, a set of analytical formulae for performance evaluation both in the power generation mode and in the cooling mode were given by Ioffe^[7], which is denoted as the classical formulae in this study. The classical formulae for the maximum coefficient of performance (COP) and cooling power in the cooling mode are given by

$$\begin{cases} COP_{\max} = \frac{T_c}{\Delta T} \frac{\sqrt{1+Z\bar{T}} - \frac{T_h}{T_c}}{\sqrt{1+Z\bar{T}} + 1}, (a) \\ Q_{c,\max} = \frac{S^2 T_c^2}{2R} - K\Delta T, (b) \end{cases} \quad (1)$$

where ΔT and $\bar{T}$ are the temperature difference and the average temperature between T_h and T_c , respectively. $Z = \frac{S^2 \sigma}{\kappa}$ is the figure of merit of the thermoelectric material, with S , σ , and κ denoting the Seebeck coefficient, the electrical conductivity, and the thermal



conductivity, respectively. R and K are the electrical resistance and thermal conductance of the module, respectively. Since $K\Delta T$ is the heat flow through the module by thermal conduction, Eq. (1b) can be rewritten in a nondimensional form as

$$\frac{q_{c,\max}}{q_{\text{cond}}} = \frac{ZT_c^2}{2\Delta T} - 1 \quad (2)$$

where $q_{c,\max}$ and q_{cond} are the cooling power density and the thermal conducting heat flux, respectively. It should be noted that the classical formulae were derived based on the constant property assumption, i.e., the material properties are independent of temperature. However, the physical properties of practical thermoelectric materials generally show strong temperature dependence, causing some problems when directly using the classical formulae.

This issue was first noticed in the power generation mode, since the temperature difference across the module could be as large as hundreds of degrees, allowing for remarkable variations of the thermoelectric properties. Even though the averaged properties are used, non-negligible errors still occur in certain cases. Many researchers have made efforts to solve this issue^[8-12]. In 2015, the authors^[13] proposed a cumulative model and defined the engineering ZT (ZT_{eng}) for thermoelectric power generation, to fully take into account the temperature dependence of material properties. Much better predictions of the maximum efficiency were achieved compared to the averaged property-adopted classical formula. Linker *et al.*^[14] also demonstrated that the accuracy of the cumulated model is comparable to that of the reduced variables approach developed by Snyder and Ursell^[15].

In the cooling mode, since the maximum temperature difference that a single-stage thermoelectric cooling (TEC) can achieve is only tens of degrees, the varying spaces for material properties are more limited than those in the power generation mode. Consequently, the classical formulae are expected to work with greater reliability. However, according to Wang *et al.*'s numerical modeling^[16], the variable material properties were observed to have significant effects on the cooling capacity and COP of TECs under high electric currents, with the maximum COP being overestimated by 17.6% by the constant property model. This indicates a need for a comprehensive analysis of the thermoelectric cooling performance with temperature dependent properties. Analytical studies by Yamashita^[17] and Ju *et al.*^[18] have already paid attention to this issue. Nevertheless, their models can only account for material properties with relatively simple temperature dependences, i.e., linear and quadratic relationships between the properties and temperature can be treated, resulting in a loss of generality to some extent.

In this work, to account for the most practical variations of material properties with temperature, we firstly extend the original cumulative model for thermoelectric power generation to the cooling mode, and derive explicit relationships between the maximum COP/cooling power and ZT_{eng} . Secondly, an estimation of the original cumulative model is performed by comparisons of the maximum COP and cooling power predicted by both the classical formulae and the original cumulative model to numerically obtained results. Eight thermoelectric materials (labeled as $n = 1-8$) with state-of-the-art ZT values near room temperature are used in the estimation, and their properties are presented in Figure 1 and Figure 2 (just to avoid busy graphs). To cover as many cases as possible, the former seven Bi_2Te_3 based materials are chosen in such a way that they show the peak ZT at different temperatures from 300 K to 450 K^[19-25], while the last one is the SnSe crystalline material reported by Liu *et al.*^[26], whose ZT is ~ 1.5 at 300 K and rises all the way to ~ 2.6 at 625 K. Since the properties of the above materials were mostly reported above 300 K, the estimated range of T_c is set to 300-350 K, which fits well with the electronics cooling applications. Following an analysis of the prediction errors, a refined cumulative model is presented to give more accurate predictions of the maximum cooling power. Finally, our model is able to predict the maximum COP within $\pm 2\%$ and the maximum cooling power within $\pm 5\%$ in a practically meaningful ΔT range of 10-70 K. The model is also applied to the commercial P and N type Bi_2Te_3 -based materials with available property data^[27] for sub-ambient cooling, and gives reasonable predictions of the maximum cooling power in a T_c range of 230-280 K and a ΔT range of 10-50 K. This work fills the gap between the classical formulae and the real-world thermoelectric cooling performance under finite temperature differences, which could be beneficial to the development and application of thermoelectric conversion technology for cooling purposes.

2. Methods

2.1 The original cumulative model

To establish a baseline for comparison, the original cumulative model must first be adapted for cooling applications. The derivation of the analytical expressions for the maximum COP and nondimensional cooling power is similar to that in our previous study for thermoelectric power generation, so they are directly written here and the detailed information can be found in SI.

$$\begin{cases} COP_{\max} = \frac{\alpha_0 \sqrt{1+\alpha_1(ZT)_{\text{eng}} - \alpha_2}}{\sqrt{1+\alpha_1(ZT)_{\text{eng}} + 1}}, & (a) \\ \frac{q_{c,\max}}{q_{\text{cond}}} = \frac{\alpha_0^2}{4W_J} (ZT)_{\text{eng}} - 1, & (b) \end{cases} \quad (3)$$

where

$$\alpha_i = \frac{T_c S(T_c)}{\int_{T_c}^{T_h} S(T) dT} + \frac{W_T \int_{T_c}^{T_h} \tau(T) dT}{\int_{T_c}^{T_h} S(T) dT} + i W_J, i = 0, 1, \text{ and } 2 \quad (4)$$

$$\begin{cases} W_J = \frac{\int_{T_c}^{T_h} \int_{T_c}^T \rho(T) dT dT}{\Delta T \int_{T_c}^{T_h} \rho(T) dT}, (a) \\ W_T = \frac{\int_{T_c}^{T_h} \int_{T_c}^T \tau(T) dT dT}{\Delta T \int_{T_c}^{T_h} \tau(T) dT}, (b) \end{cases} \quad (5)$$

and

$$(ZT)_{\text{eng}} = Z_{\text{eng}} \Delta T = \frac{\left(\int_{T_c}^{T_h} S(T) dT \right)^2}{\int_{T_c}^{T_h} \rho(T) dT \int_{T_c}^{T_h} \kappa(T) dT} \Delta T \quad (6)$$

The expressions in Eq. (3) are in similar forms with those in Eq. (1a) and Eq. (2), and the $(ZT)_{\text{eng}}$ as well as the involved coefficients defined by Eqs. (4-5) can be obtained by integrations of the material properties within the operating temperature range.

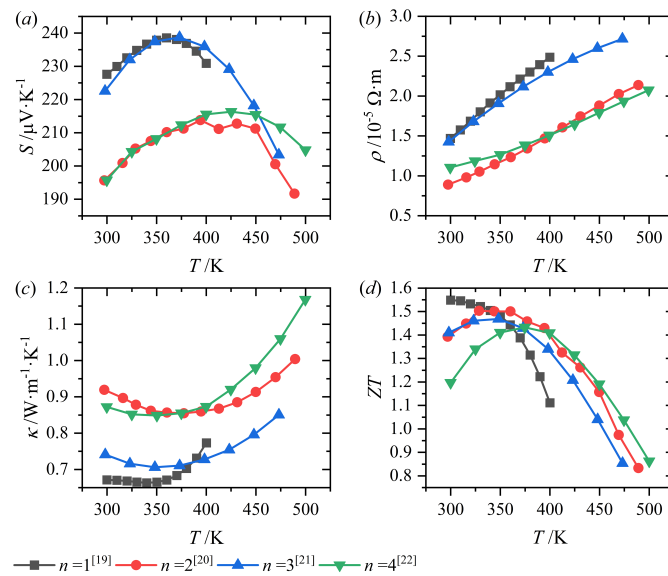


Figure 1. Physical properties of the studied thermoelectric materials ($n = 1-4$). (a) Seebeck coefficient; (b) Electrical resistivity; (c) Thermal conductivity; (d) ZT value.

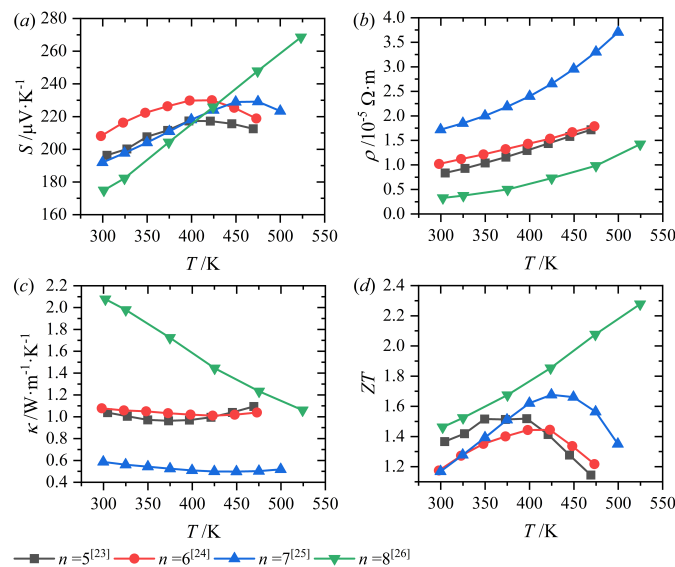


Figure 2. Physical properties of the studied thermoelectric materials ($n = 5-8$). (a) Seebeck coefficient; (b) Electrical resistivity; (c) Thermal conductivity; (d) ZT value.

2.2 Validation of the model

To obtain the numerical reference solutions, the complete governing equations of the thermoelectric element were solved iteratively with temperature-dependent material properties. Starting from an initial temperature distribution, the local thermoelectric properties were updated according to the calculated temperature field, and the governing equations were repeatedly solved until convergence was achieved. The converged numerical results for the temperature distribution, maximum COP, and cooling power were then used as benchmark solutions for validating the proposed analytical models. The predicted maximum COP and cooling power by both the classical formulae (in which the integral average value of Z , $Z_{\text{int}} = \frac{1}{\Delta T} \int_{T_c}^{T_h} Z(T) dT$, is used) and the original cumulative model are compared to numerical results within a ΔT range of 10–50 K and T_c range of 300–350 K, where the hot-side temperature is determined by $T_h = T_c + \Delta T$, as illustrated in Figure 3 and Figure 4 for materials 1 to 4. The comparison results for materials 5 to 8 are shown in Figures S1,S2.

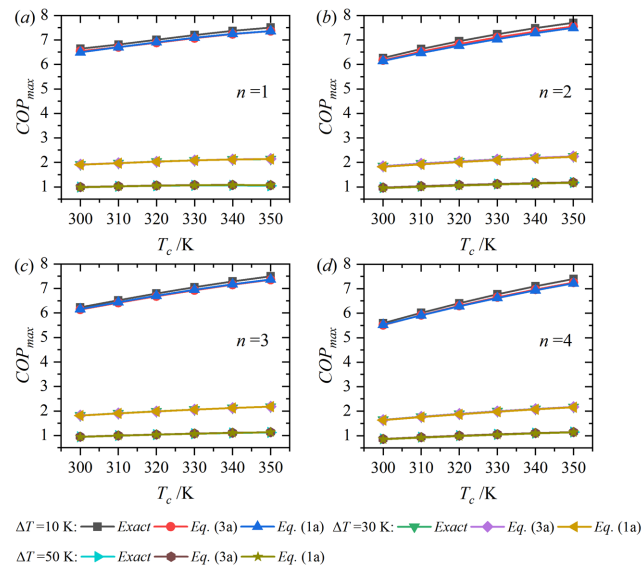


Figure 3. Comparisons of predicted maximum COP by Eqs. (3a) and (1a) to numerical results ($n = 1-4$). COP: coefficient of performance.

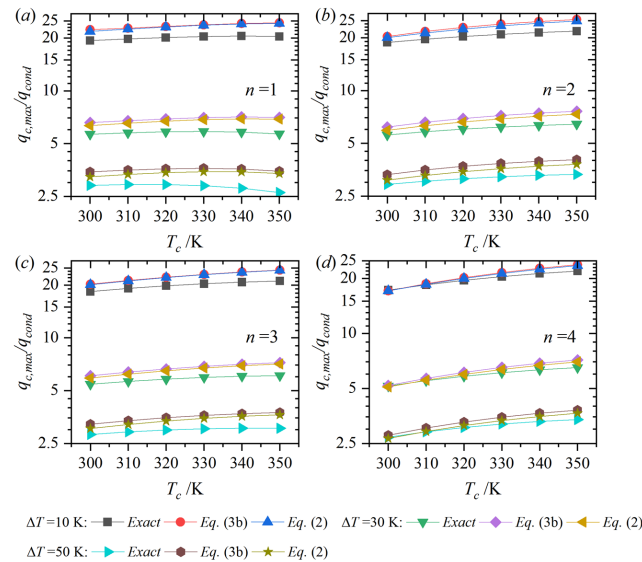


Figure 4. Comparisons of predicted maximum nondimensional cooling power density by Eqs. (3b) and (2) to numerical results ($n = 1-4$).

The results demonstrate that the maximum COP is closely predicted by both the classical formula given by Eq. (1a) and the original cumulative model given by Eq. (3a) for all of the eight materials in the current ΔT and T_c ranges. However, the predictions of the maximum nondimensional cooling power density by both Eq. (2) and Eq. (3b) exhibit discrepancies as shown in Figure 4 and Figure S2. Even if ΔT is as low as 10 K, the average error of Eq. (3b) reaches $\sim 17\%$ for the first material ($n = 1$). The error bands of the maximum

COP by Eq. (3a) and cooling power by Eq. (3b) are further shown in Figure 5. While the cumulative model provides reliable prediction for the maximum COP, it exhibits significant discrepancies in estimating the cooling power density with errors exceeding 30%. Consequently, the cumulative model offers no significant advantage over classical formulae for evaluating thermoelectric cooling performance. Therefore, to enhance the reliability of cooling power analysis, a refined model is essential, following a thorough investigation into the sources of error in the original cumulative framework.

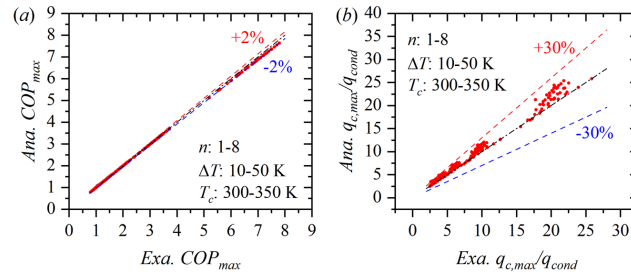


Figure 5. Error bands for predictions of (a) maximum COP by Eq. (3a); (b) Nondimensional cooling power density by Eq. (3b). COP: coefficient of performance.

3. Results and Discussion

3.1 Analysis of the prediction error

To investigate the large prediction errors of the maximum cooling power by Eq. (3b), it is necessary to revisit the derivation process (SI). The primary assumption made is the linear temperature distribution when converting the variables of the single and double integrals from x to T , as shown in Eqs. (S9,S10,S11). To check the rationality of the above assumption, the temperature distribution within the thermoelectric element is extracted from the numerical results for both the maximum COP case and the maximum cooling power case, as illustrated in Figure 6a. It is obviously seen that the temperature distributions in the maximum COP cases under various ΔT well support the linear assumption, explaining the good predictions of the maximum COP by Eq. (3a).

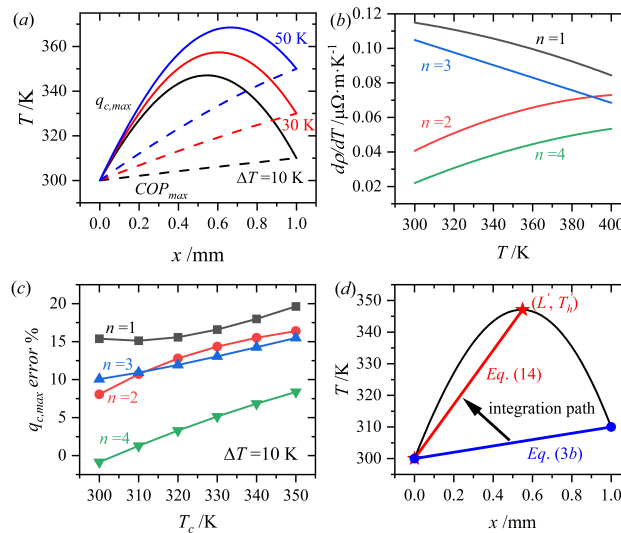


Figure 6. (a) Temperature profiles in maximum COP and cooling power modes under different ΔT ; (b) Varying rates of electric resistivities with respect to temperature for materials $n = 1-4$; (c) Prediction errors of maximum cooling power for materials $n = 1-4$ ($\Delta T = 10$ K); (d) Change of integration path in the refined model. COP: coefficient of performance.

Nevertheless, the temperature distributions severely deviate from the linear assumption in all of the maximum cooling power cases. Taking $\Delta T = 10$ K as an example, although the terminal temperatures are 300 K and 310 K, respectively, the highest temperature within the element approaches 350 K at the central part of the element. This kind of convex temperature distribution was also noticed by Wang *et al.*^[16] under relatively large electric currents, which is supposed to be the primary reason for the inaccurate predictions of the maximum cooling power. In the case of $\Delta T = 10$ K, the electric resistivity data between 300 K and 350 K should be considered for an accurate evaluation of the element's electric resistance. However, only those between 300 K and 310 K are included in Eq. (S9) under the assumption of linear temperature distribution. Similarly, the double integrals in Eqs. (S10) and (S11) must also incorporate the property data between 310 K and 350 K. Consequently, some of the constituent terms in the cooling power density analysis are significantly miscalculated due to the assumption. Based on this, one can expect a direct correlation between the

prediction errors of the maximum cooling power and the material's temperature dependence of electric resistivity, i.e., a material with larger varying rates of the electric resistivity with temperature generally yields larger errors. Figure 6b presents the varying rates of the electric resistivities of the former four materials from 300 K to 400 K, and Figure 6c shows the prediction errors of the maximum nondimensional cooling power density for these materials in case of $\Delta T = 10$ K. A positive correlation is clearly seen between the level of electric resistivity's varying rate and that of the prediction error among the four materials, which convincingly supports the previous explanation. Therefore, to accurately evaluate the maximum cooling power between T_c and T_h , it is necessary to account for comprehensive material property data, rather than limited property data just between T_c and T_h .

3.2 Refined cumulative model

Based on the above discussion, a possible way to refine the original cumulative model is proposed. As illustrated in Figure 6d, by changing the integration path of the governing equation (Eq. (S1)) from $0 \rightarrow L$ to $0 \rightarrow L'$, with L' denoting the location of the highest temperature T'_h , the deviation of the local temperature curve from a linear one is reduced, and all of the property data between T_c and T'_h are involved in the integration. By doing so, the prediction capability of the maximum cooling power is expected to be enhanced. Figure 7 provides a schematic illustration of the heat-flow partition of Joule and Thomson heat under a convex temperature profile, which offers a physical interpretation for the modified integration path adopted in the revised cumulative model.

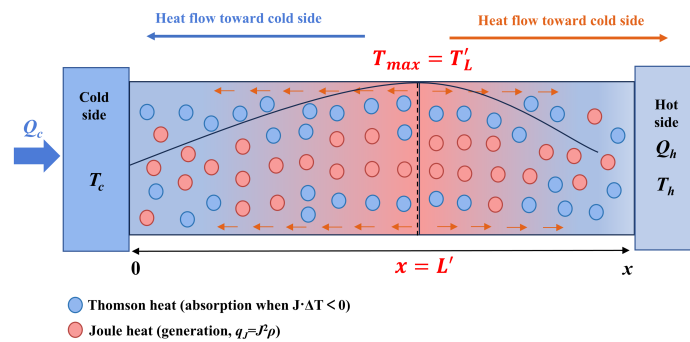


Figure 7. Schematic illustration of heat-flow partition of Joule and Thomson heat under a convex temperature profile.

Before implementing the refinement, the specific information of the highest temperature point needs to be addressed to serve as the upper integration boundary. However, since the location coordinate and the value of the highest temperature are implicit solutions to the governing equation, they are not explicitly available. As an approximation, values obtained from the constant-property governing equation are utilized,

$$\begin{cases} L' = \left(\frac{1}{2} + \frac{\Delta T}{2T_c} \right) L, & \text{(a)} \\ T'_h = \frac{1}{2} Z T_c^2 \left(\frac{1}{2} + \frac{\Delta T}{2T_c} \right)^2 + T_c, & \text{(b)} \end{cases} \quad (7)$$

and the cumulative $Z = \frac{\left(\int_{T_c}^{T_h} S(T) dT \right)^2}{\int_{T_c}^{T_h} \rho(T) dT \int_{T_c}^{T_h} \kappa(T) dT}$ is used to calculate L' and T'_h . The refined cumulative model is reconstructed in the following manner.

Consistent with the original cumulative model, the governing equation is integrated once, resulting in Eq. (S2), which is given here for the convenience of reading,

$$\kappa(T) \frac{dT}{dx} + J^2 \int_0^x \rho(T) dx - J \int_{T_c}^T \tau(T) dT + C_1 = 0 \quad (S2)$$

The refined model will depart from the original one hereafter. Note that the temperature gradient at the highest temperature point is zero, i.e., $\left. \frac{dT}{dx} \right|_{x=L'} = 0$. Substituting this boundary condition into Eq. (S2), C_1 is given by

$$C_1 = -J^2 \int_0^{L'} \rho dx + J \int_{T_c}^{T'_h} \tau dT \quad (8)$$

The cooling power density at the cold side is re-obtained as

$$q_c = S(T_c) T_c J + C_1 = -J^2 \int_0^{L'} \rho dx + \left[S(T_c) T_c + \int_{T_c}^{T'_h} \tau dT \right] J \quad (9)$$

And the maximum cooling power density is reached as

$$q_{c,\max} = \frac{\left[S(T_c)T_c + \int_{T_c}^{T'_h} \tau dT \right]^2}{4 \int_0^{L'} \rho dx} \quad (10)$$

when

$$J = \frac{\left[S(T_c)T_c + \int_{T_c}^{T'_h} \tau dT \right]}{2 \int_0^{L'} \rho dx} \quad (11)$$

Applying the linearized assumption to the integral term, i.e., $\int_0^{L'} \rho dx \approx \frac{L'}{\Delta T'} \int_{T_c}^{T'_h} \rho dT$, and letting

$$\beta_0 = \frac{S(T_c)T_c + \int_{T_c}^{T'_h} \tau dT}{\int_{T_c}^{T'_h} S dT} \quad (12)$$

Eq. (10) is rewritten as

$$q_{c,\max} = \frac{\beta_0^2 \left(\int_{T_c}^{T'_h} S dT \right)^2 \Delta T'}{4 \int_{T_c}^{T'_h} \rho dT \int_{T_c}^{T'_h} \kappa dT} \frac{1}{L'} \int_{T_c}^{T'_h} \kappa dT \quad (13)$$

The term $\frac{1}{L'} \int_{T_c}^{T'_h} \kappa dT$ in Eq. (13) is again the thermal conducting heat flux, so the nondimensional cooling power density is

$$\frac{q_{c,\max}}{q_{\text{cond}}} = \frac{\beta_0^2 \left(\int_{T_c}^{T'_h} S dT \right)^2 \Delta T'}{4 \int_{T_c}^{T'_h} \rho dT \int_{T_c}^{T'_h} \kappa dT} = \frac{\beta_0^2}{4} Z'_{\text{eng}} \Delta T' = \frac{\beta_0^2}{4} (ZT)'_{\text{eng}} \quad (14)$$

And the corresponding electric current density is

$$J_{q_{c,\max}} = \frac{\beta_0 \int_{T_c}^{T'_h} S dT}{2 \frac{L'}{\Delta T'} \int_{T_c}^{T'_h} \rho dT} \quad (15)$$

Figure 8 and Figure S3 present comparisons of the maximum nondimensional cooling power densities obtained by Eqs. (12) and (14) to exact data by numerical iterations. Meanwhile, the error band for all of the eight materials is shown in Figure 9a with an upper limit of 20%. While the refined cumulative model demonstrates more reliable prediction, there is still substantial room for additional refinement. To further improve its accuracy, it is essential to inspect the potential error sources.

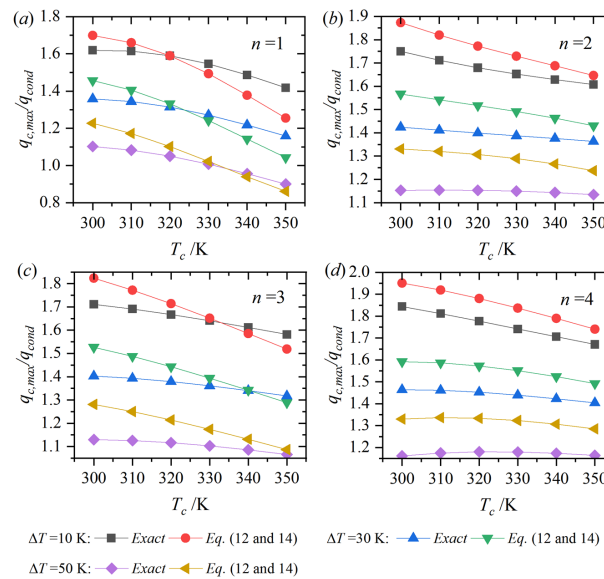


Figure 8. Comparisons of predicted maximum nondimensional cooling power density by Eqs. (12) and (14) to numerical results ($n = 1-4$).

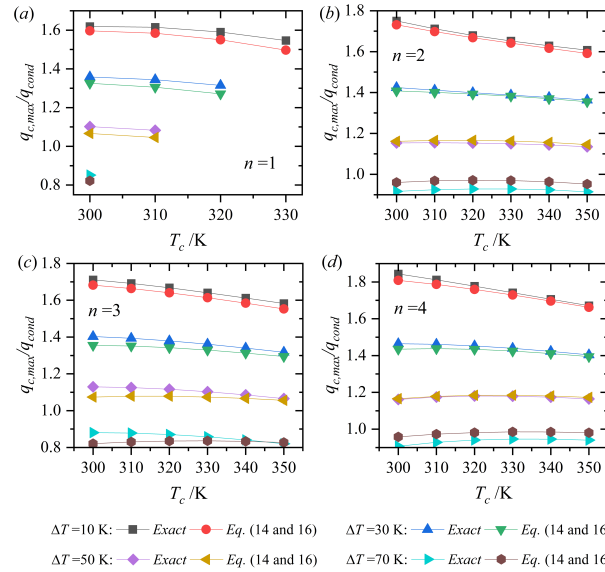


Figure 9. Error bands for predictions of maximum nondimensional cooling power density by (a) Eqs. (12) and (14); (b) Eqs. (16) and (17) in the modified model.

There are two major error sources in the refined model. The first one is the highest temperature point (L' , T'_h) given by Eq. (7) based on the constant-property governing equation. In the context of the constant-property model, it was reported by Fraisse *et al.* that the Thomson effect should not be taken into account when the Seebeck coefficient is assumed to be constant, or it will cause additional errors^[41]. Considering the constant-property based (L' , T'_h), a partial suppression of the Thomson effect could be beneficial, which is reflected in β_0 by Eq. (12). Guided by this thought, a constant coefficient smaller than 1 is introduced to the second term in the numerator of β_0 . Through an empirical optimization to minimize residual errors, a suppression coefficient of 0.5 was determined to yield the highest predictive accuracy, and

$$\beta_0 = \frac{S(T_c)T_c + 0.5 \int_{T_c}^{T'_h} \tau dT}{\int_{T_c}^{T'_h} S dT} \quad (16)$$

is used instead of Eq. (12) hereafter. The second one is the approximation of $\int_0^{L'} \rho dx \approx \frac{L'}{\Delta T} \int_{T_c}^{T'_h} \rho dT$, which is valid only if the temperature distribution between T_c and T'_h is linear. Looking back to Figure 6d, there still exists departure of the actual temperature distribution from the new integration path, especially near the T'_h point. After a careful comparison between $\int_0^{L'} \rho dx$ and $\frac{L'}{\Delta T} \int_{T_c}^{T'_h} \rho dT$ (see SI for more details), an additional term is introduced into Eq. (14) to obtain the following equation,

$$\frac{q_{c,max}}{q_{cond}} = \frac{\beta_0^2}{4} (ZT)'_{eng} - 0.6 \frac{\rho'(T_c)}{\rho'(T_h)} \frac{\Delta T}{T_c} \quad (17)$$

where $\rho'(T_c) = \left. \frac{d\rho}{dT} \right|_{T=T_c}$ and $\rho'(T_h) = \left. \frac{d\rho}{dT} \right|_{T=T_h}$ account for the variation characteristics of the electric resistivity across the temperature range from T_c to T_h .

Figure 10 and Figure S4 present comparisons of the maximum nondimensional cooling power density predicted by Eqs. (16) and (17) to numerical data. The further refined cumulative model demonstrates significantly higher accuracy relative to the previous models. Figure 9b confirms that all data points for eight materials fall within a $\pm 5\%$ margin of error over an extended ΔT range of 10–70 K. It should be noted that this error margin refers to the relative deviation with respect to the corresponding numerical reference solutions obtained using the same temperature-dependent material properties reported in the literature, rather than the absolute prediction accuracy for practical thermoelectric devices.

In addition to the numerical validation, the proposed analytical model was further evaluated using the performance specifications of a commercial thermoelectric module (Thermonamic Electronics (Jiangxi) Corp.)^[28], as shown in Figure 11. Under the representative operating condition of $\Delta T = 30$ K and $T_h = 323$ K, the predicted optimal operating current is 4.8 A, identical to the manufacturer's test specified value. The predicted maximum cooling capacity is 45.3 W, in excellent agreement with the specified value of 45.5 W, with a relative deviation of only 0.4%. These results demonstrate the capability of the proposed analytical model to accurately predict the performance of practical p–n thermoelectric cooling modules.

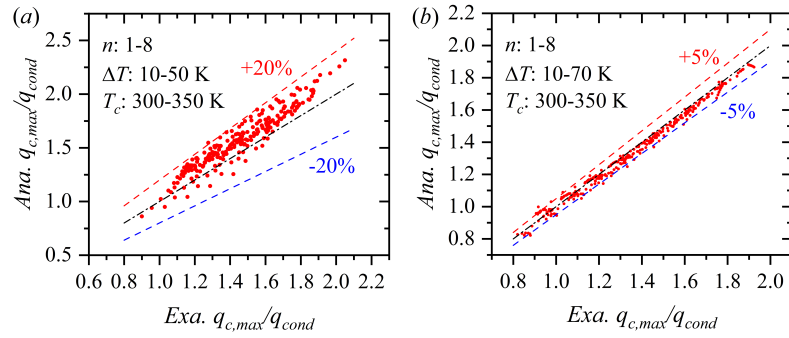


Figure 10. Comparisons of predicted maximum nondimensional cooling power density by Eqs. (16) and (17) to numerical results ($n = 1-4$).

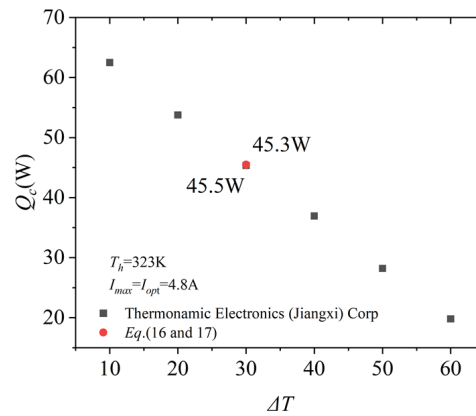


Figure 11. Comparison between the predicted maximum cooling capacity and the commercial TEC performance at $T_h = 323$ K. TEC: thermoelectric cooling.

Eqs. (16) and (17) are further applied to commercial P and N type Bi_2Te_3 based thermoelectric (TE) materials for sub-ambient cooling power evaluations, and the analytically predicted results as well as numerical results are compared in Figure 12, covering a T_c range of 230–280 K and a ΔT range of 10–50 K. The thermoelectric properties of these two materials between 200 K and 350 K are illustrated in Figure S5. The analytical model shows a strong correlation with numerical results, especially for cases with nondimensional cooling power densities above 0.8. It is concluded that the analytical model proposed in this work is universal across various TE materials and application scenarios.

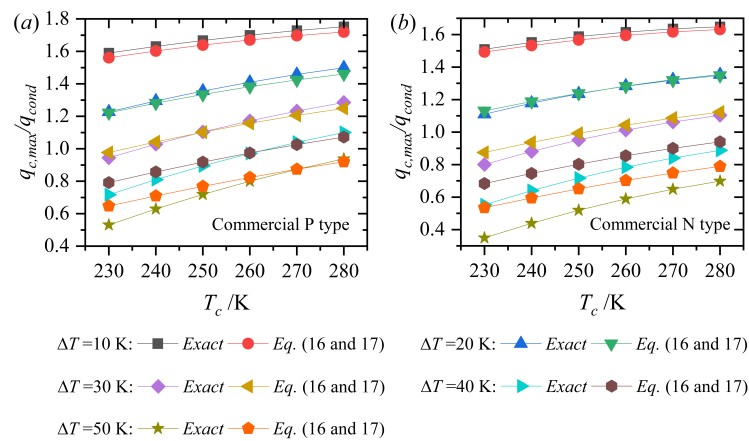


Figure 12. Comparisons of predicted maximum nondimensional cooling power density by Eqs. (16) and (17) to numerical results for sub-ambient cooling with commercial P and N type materials.

4. Conclusions

This study presents a refined cumulative model that provides a robust and accurate framework for evaluating real-world thermoelectric cooling performance under finite temperature differences. By utilizing a one-step integration method with the upper

boundary scheme, the proposed model successfully addresses the challenges posed by temperature-dependent material properties and inaccuracy from non-linear temperature profiles within the thermoelectric element. The advantage of this model is evidenced by its significant performance in assessing maximum cooling performance within $\pm 5\%$ across a temperature difference range of 10–70 K for various materials. This represents a substantial improvement over previous approaches including the classical and the original cumulative models, which failed to yield reliable cooling power estimations due to the assumption of constant material properties or a linear inner temperature distribution. Consequently, this work offers an elegant and widely applicable solution for evaluating real-world thermoelectric cooling performance, and is expected to facilitate the design, development, and application of thermoelectric cooling technologies.

Supplementary materials

The supplementary material for this article is available at: [Supplementary materials](#).

Declarations

Authors contribution

Zhu K, Kim H: Conceptualization, formal analysis, investigation, writing-original draft, funding acquisition.

Li X, Brown A, Shi R: Formal analysis, writing-review & editing.

Liu W: Supervision, conceptualization, methodology, writing-review & editing, funding acquisition.

Conflicts of interest

Weishu Liu is an Editorial Board Member of *Thermo-X*. The other authors declare no conflicts of interest.

Ethical approval

Not applicable.

Consent to participate

Not applicable.

Consent for publication

Not applicable.

Availability of data and materials

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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